

# Parameterization of atmospheric turbulence

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## Contents (“wish list”, most, not all in fact included ... )

- Turbulence? Turbulence transports / methods used;
- Turbulence closure. Example: Mellor-Yamada (MY) 2.5;
- Realizability and master length scale issues of the MY 2.5;
- The surface layer: Monin-Obukhov (MO) similarity;
- MO issues: free convection / the Beljaars correction, the Zilitinkevich extension for the stable case, BL “structures”;
- The molecular sublayer;
- Horizontal diffusion: do we need it, if so why / what is it?

# Turbulence?

**Exists** (an observed fact!)

Results in important transports

Transports:

- *Advection*/ transports by model-resolved motions;
- *Turbulence* transports: transports by turbulence eddies;
- Near the ground surface: *molecular* transports

## 1. Turbulence transports / methods used: \*

*Horizontal* turbulence transports are unimportant / negligible with resolved horizontal scales much larger than the vertical scale - as is the case in regional atmospheric models (e.g., Mellor 1985). Or, Bougeault (1997, p 79): three-dimensional turbulence effects "become important only when the horizontal resolution approaches 1 km".

Thus: *vertical* transports. The typical approach: for a specific variable  $A$  (quantity per unit volume:  $\rho A$ ), define "exchange coefficient"  $K_A$  by

$$\frac{\partial A}{\partial t} = \frac{\partial}{\partial z} \left( K_A \frac{\partial A}{\partial z} \right) \quad (1)$$

Fundamental task: determine  $K_A$ .

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\* Scanned pieces from an ICTP lecture, 1999 (or 2000?): "Boundary layer, turbulence transports, horizontal diffusion", updated

Very large variety of schemes. Some recent references: Bougeault (1997); reports which follow Nielsen (1999).

- Schemes expressing  $K_A$  as a function of model-resolved variables (shear, buoyancy / Richardson number, ...) Examples: "Louis" scheme, "Holtslag" scheme, ... (e.g., Louis 1979, Rummukainen 1999, ...)
- Schemes with a prognostic equation for the turbulence kinetic energy (TKE). Popular member: Mellor-Yamada level 2.5
- Schemes with two and more prognostic equations for turbulence quantities. E.g., TKE-e schemes, Mellor-Yamada level 3 or more, ...

Also: non-local schemes, not following (1), or not only following (1).

Turbulence closure:

- Consider variables consisting of **mean values**, and fluctuations,

e.g.: 
$$\tilde{u} = U + u, \quad \tilde{v} = V + v, \quad \dots, \quad (2)$$

- Assume** a set of properties for ensemble averaging ("Reynolds averaging"):

$$\overline{AB} = A\overline{B} + \overline{a'b'} \quad \overline{\overline{A}} = \overline{A} \quad \dots \quad (3)$$

- Assume** that the **mean values** satisfy the governing equations; write them  $\rightarrow$  (4)
- Write governing equations for the total values:  $\rightarrow$  (5)
- Subtract (4) from (5), to obtain prognostic equations for  $u, v, \dots$  (In tensor notation:  $u_j$ , and  $\theta$ )  $\rightarrow$  (6).

New variables appear as a result of (3):

$$\overline{u_i u_j} \quad \overline{u_j \theta} \quad (7)$$

"Reynolds stresses". These are the variables we need to describe the effect of turbulence on mean quantities. However, more variables than prognostic equations: *the closure problem*.

Get prognostic equations for Reynolds stresses by time differentiating (7) and inserting from (6).

However, yet additional new variables:

$$\overline{u_k u_i u_j} \quad \overline{p u_j} \quad \overline{u_i u_j \theta} \quad \overline{p \theta}$$

A variety of assumptions by numerous people.



Mellor and Yamada (1974, 1982):

Assumptions due to Kolmogorov, 1942, and Rotta, 1951. Tensor symmetry properties, dimensional analysis considerations. Analyze terms with respect to order of deviation from isotropy. Introduce systematic simplifications based on the assumption that the degree of anisotropy is small.

---> terms including a variety (five) of length scales

Assumption: all five length scales proportional to a single "*master length scale*",  $l$

Mellor-Yamada "level 2.5": reduce the problem to just one prognostic equation ("M-Y 2.5"); very popular – many models



$$\frac{d(q^2/2)}{dt} - \frac{\partial}{\partial z} (lqS_q \frac{\partial}{\partial z} (\frac{q^2}{2})) = P_s + P_b - \epsilon,$$

where

$$q^2 = u^2 + v^2 + w^2,$$

is twice the turbulence kinetic energy.  $P_s$  and  $P_b$ : shear and buoyancy production, given by

$$P_s = - \overline{wu} \frac{\partial U}{\partial z} - \overline{wv} \frac{\partial V}{\partial z},$$

$$P_b = \beta g \overline{w\theta_v};$$

Dissipation,  $\varepsilon$ , is given by

$$\varepsilon = \frac{q^3}{B_1 l}.$$

$S_q$ ,  $\beta$ , and  $B_1$  above are constants. Exchange coefficients for momentum and heat,  $K_M$  and  $K_H$ , are

$$K_M = lq S_M, \quad K_H = lq S_H.$$

The "stability functions"  $S_M$  and  $S_H$  can be calculated from

$$G_M = \frac{l^2}{q^2} \left[ \left( \frac{\partial U}{\partial z} \right)^2 + \left( \frac{\partial V}{\partial z} \right)^2 \right],$$

$$G_H = -\frac{l^2}{q^2} \beta g \frac{\partial \Theta_v}{\partial z},$$

via equations that involve additional constants (Mellor and Yamada 1974, 1982; also [Janjic 1990](#)).

The realizability problem: solving for  $S_M$  and  $S_H$  -- ill-conditioned in a region of the  $G_M, G_H$  plane. Janjic (1990, similar to MY 1982):

$$G_H \leq 0.024, \quad G_M \leq 0.036 - 15 G_H.$$

Struggling: Helfand and Labraga (JAS, 1988); Galperin et al. (JAS 1988: level 2  $1/4$  scheme).

In the Eta: rather than restrict  $G_H, G_M$ , **restrict  $l$**

Mesinger (1993a), Janjic (1996)

A summary of the problem described in Section 4 of

Mesinger, F., 2010: Several PBL parameterization lessons arrived at running an NWP model. Intern. Conf. Planetary Boundary Layer and Climate Change, IOP Publishing, IOP Conf. Series: Earth and Environmental Science **13** (2010) 012005 doi: 10.1088/1755-1315/13/1/012005. (Available online at

<http://iopscience.iop.org/1755-1315/13/1/012005>). No charge ☺ !!

How was this discovered ? Mellor-Yamada clipping:

$$G_H \leq 0.024, \quad G_M \leq 0.36 - 15G_H$$

Recall:

$$G_M \equiv \left(\frac{l}{q}\right)^2 \left[ \left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2 \right], \quad G_H \equiv -\left(\frac{l}{q}\right)^2 \beta g \frac{\partial \Theta_v}{\partial z}$$

and:

$$P_s + P_b - \varepsilon = \frac{q^3}{l} \left( S_M G_M + S_H G_H - \frac{1}{B_1} \right)$$

with the Blackadar length scheme

$$l = l_0 k z / (k z + l_0), \quad l_0 = \alpha \int z q dp / \int q dp, \quad \alpha = \text{const}$$

but the more recent code had above the boundary layer

$$l = \min(l_\Delta, l_D, k z), \quad l_\Delta = c_s \Delta z \quad l_D = c_r q / N$$

Blackadar scheme all the way into the upper troposphere:

The model wanted to generate turbulence, but the  $G_M, G_H$  were apparently clipped and on top of it the TKE total production was being divided by a large Blackadar  $l$  !!

No turbulence above the boundary layer :(

Switching to the above PBL scheme (bottom line of the previous slide): nicely looking upper troposphere turbulence ! :)

Serendipity ?

#### 4. The surface layer: Monin-Obukhov similarity

"Surface layer": shallow layer where *the turbulent fluxes differ little from their surface value*. Extending from the ground to some meters above. ("Some meters": 5 to 50 m)

Also "constant flux layer". Warning: this is precisely the layer in which the turbulent fluxes change most rapidly!

"Atmospheric surface layer", ASL

Basic notation. Consider the "neutral" case first: heat transport not having a significant impact. (Always near the surface).

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Relevant variables: height,  $z$ , and "friction velocity",  $u_*$ , defined by

$$u_* u_* = - \overline{u'w'}.$$

Also: "velocity scale".

Momentum profile:

$$\frac{du}{u_*} = \frac{dz}{l}$$

where

$$l = kz$$

is a characteristic length scale, or eddy size.  $k$ : von Karman constant, 0.4. For traditional reasons, we have here changed notation to use lower case  $u$  for the mean velocity. Integration leads to

$$u = \frac{u_*}{k} \ln \frac{z}{z_0}. \quad (8)$$

"The logarithmic wind profile".  $z_0$ : "roughness length". However: "roughness length for momentum",  $z_{0u}$ , better.



Stratification (Monin-Obukhov):

Sensible heat flux,  $-\overline{w'\theta'}$ , is relevant. Traditional: define "temperature scale",  $\theta_*$ , by

$$\theta_* u_* = -\overline{\theta'w'}.$$

Using this temperature scale (in fact, sensible heat flux), "Monin-Obukhov" (MO) length is defined, e.g., by

$$L = \frac{u_*^2 \Theta}{kg \theta_*}. \quad (9)$$

Nondimensional height can now be formed,  $z/L$ , and instead of the

$\frac{du}{dz} = \frac{u_*}{kz}$ , etc., MO similarity states that

$$\frac{du}{dz} = \frac{u_*}{kz} \Phi_u\left(\frac{z}{L}\right), \quad \frac{d\Theta}{dz} = \frac{\theta_*}{kz} \Phi_\theta\left(\frac{z}{L}\right), \quad (10)$$

etc.  $\Phi_u, \Phi_\theta, \dots$ : functions obtained from measurements. "Empirical functions".

To compute fluxes, we need the exchange coefficients  $K_M$  and  $K_H$ , defined by

$$-\overline{u'w'} = K_M \frac{du}{dz}, \quad -\overline{\theta'w'} = K_H \frac{d\Theta}{dz} \quad (11)$$

Solving (10) not straightforward. Highly implicit. Note:  $L$  is a function of the momentum and sensible heat flux, given by  $K_M$  and  $K_H$ , which we want to obtain. However: standard methods. Two elevations needed;

usually, iterations to solve (10), started with first guess fluxes. First guess fluxes obtained from first guess "bulk" exchange coefficients (coefficients of the finite difference forms of (11)). From the first guess fluxes, obtain first guess  $L$ . Several iterations.

What about very close to the ground?

Molecular transports take over !!

In the Eta:

Different over land (and ice) and over water

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elements:

Zilitinkevich (1995):

$$z_{0T} = z_{0m} e^{-A_0 \sqrt{u_* z_{0m} / \nu}}$$

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Over water:  
Molecular sublayer

Liu, Katsaros, Businger  
(1979, "LKB");

Janjic (1994);

also: "An upgraded  
version of the Eta model"

## Molecular sublayer:

according to measurements of Mangarella et al. (1973):

Three regimes: smooth, rough, and rough with spray;

The flow switches from one to the other according to the value of “roughness Reynolds number”,  $Rr$

$$Rr = u_* z_0 / \nu$$

Seems to work well, an example:



Typhoon  
Yancy  
(Aug. 1990)

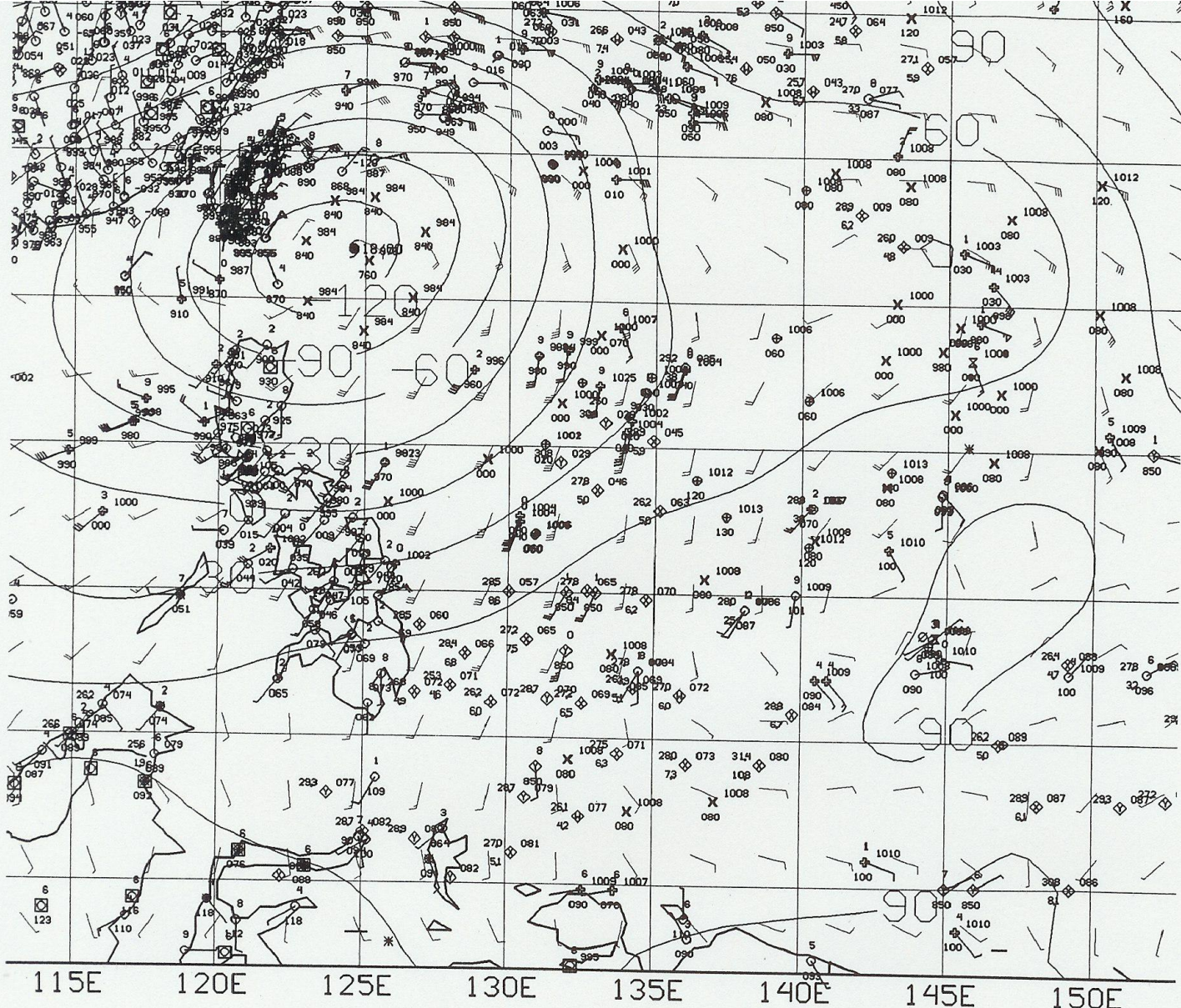
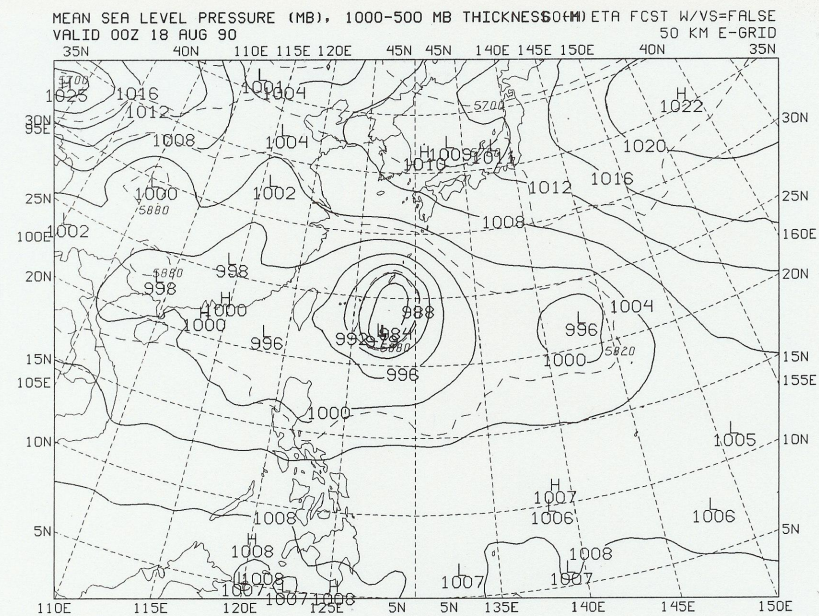


Figure 3. A section of the NMC TCM-90 surface analysis valid 0000 UTC 18 August 1990 (Courtesy of Eric Rogers). Contours of analyzed 1000 mb geopotential heights, in meters, and winds, in knots, are shown; as well as various observations.



Without  
molecular sublayer:



With  
molecular sublayer:

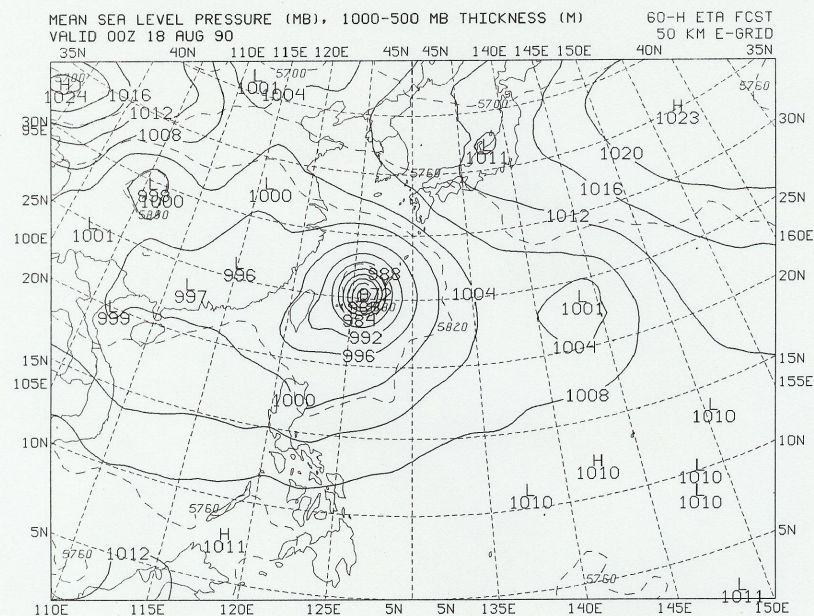


Figure 4. The Eta Model 60-h simulation of the sea level pressure, millibars, and 1000-500 mb thickness, meters, valid 0000 UTC 18 August 1990, with no parameterization of the molecular sublayer, upper panel; same except for the parameterization of the molecular sublayer being included, lower panel. (Courtesy of Eric Rogers.)

"What have you done for me lately?"

$$\begin{aligned}
\nu \frac{U_1 - U_s}{z_{1u}} &= u_* u_*, \\
\kappa \frac{\Theta_1 - \Theta_s}{z_{1\theta}} &= \theta_* u_*, \\
\varepsilon \frac{q_1 - q_s}{z_{1q}} &= q_* u_*,
\end{aligned} \tag{8.1}$$

where  $\nu$ ,  $\kappa$ , and  $\varepsilon$  are the kinematic viscosity, thermal diffusivity, and molecular diffusivity of water vapor, respectively;  $u_*$  is the friction velocity, and  $\theta_*$  and  $q_*$  are analogously defined scaling parameters for the sensible heat and moisture fluxes, respectively. The right hand sides of (8.1) can also be expressed in terms of the standard surface layer bulk relationships, and the equations thus obtained solved for  $U_1$ ,  $\Theta_1$  and  $q_1$  provided sublayer thicknesses are known. These were obtained by Janjic by postulating

$$\frac{z_{1u} u_*}{C\nu} = \frac{z_{1\theta} u_*}{S\kappa} = \frac{z_{1q} u_*}{D\varepsilon} = \xi, \tag{8.2}$$

It was considered by Janjic adequate to keep  $\zeta$  a constant. For  $Rr \approx 1$  one obtains

$$\zeta = 0.35 \quad (29)$$

Used in the “standard” Eta

As opposed to having  $\zeta$  constant, a relationship resulting from experimental data (Brutsaert 1982, Fig. 4.1) can be used:

A question can be asked: if the linear profile at the bottom of the viscous sublayer is linearly extrapolated upwards, and the logarithmic profile of the surface layer is at the same time logarithmically extrapolated downwards, at what elevation will the two extrapolated profiles intersect? This should be the appropriate value of  $z_{1u}$ , from which  $\zeta$  can be calculated.

One obtains

$$\zeta = 11 / (M Rr^{1/4})$$

The model knows what is  $Rr$  :

Relation originally due to Charnock widely used; in the Eta:

$$z_0 = \frac{0.11\nu}{u_*} + \frac{0.018u_*^2}{g}$$

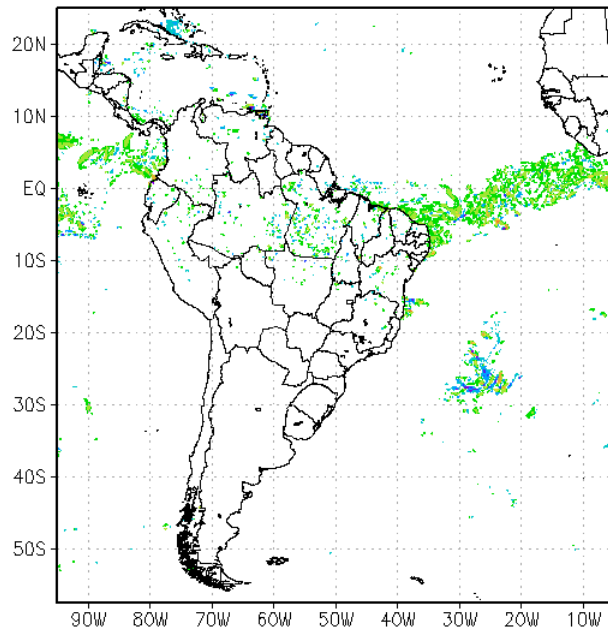
0.018: *the Charnock constant*; for further reading see e.g., Garratt (1992, pp. 98-100).

Thus,  $\zeta$  can be calculated as a function of  $Rr = u_* z_0 / \nu$   
using the Brutsaert relation (previous slide)

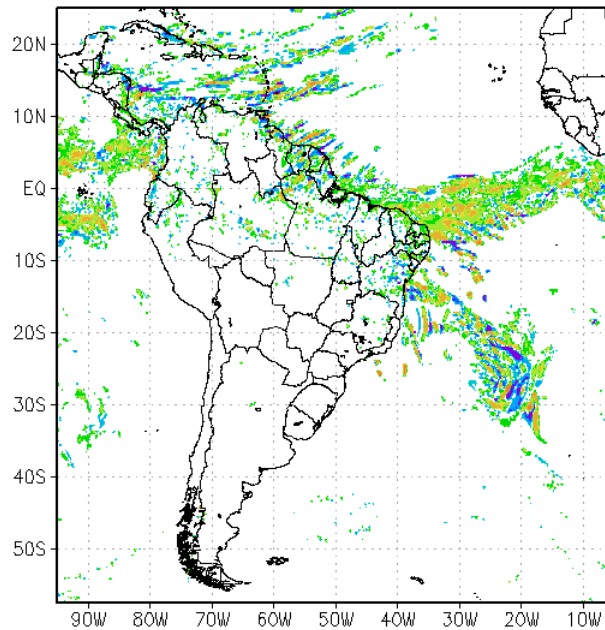
Experiments done by **Josiane Bustamante**, CPTEC:



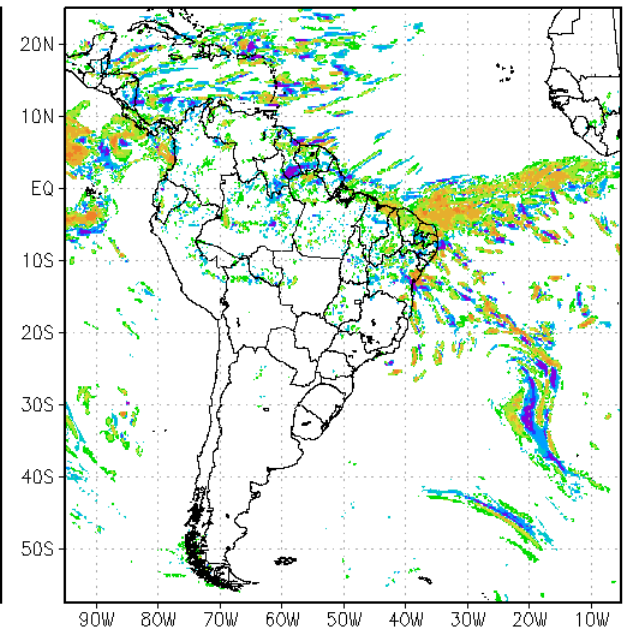
cntrl-cvisc T+24h



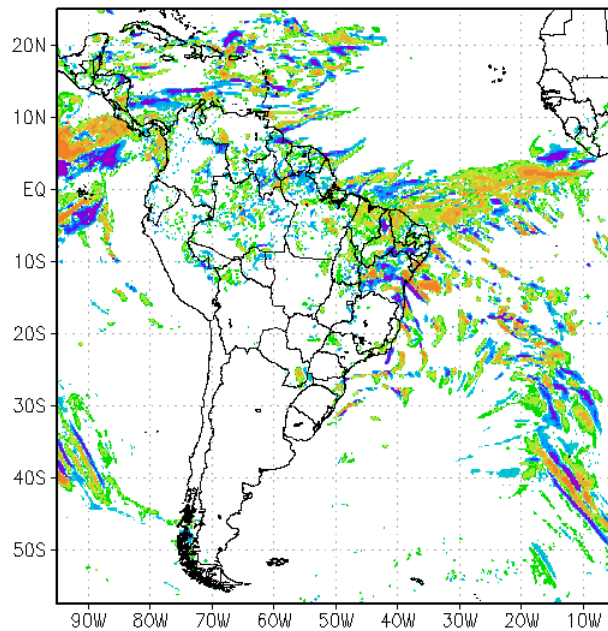
cntrl-cvisc T+48h



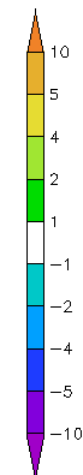
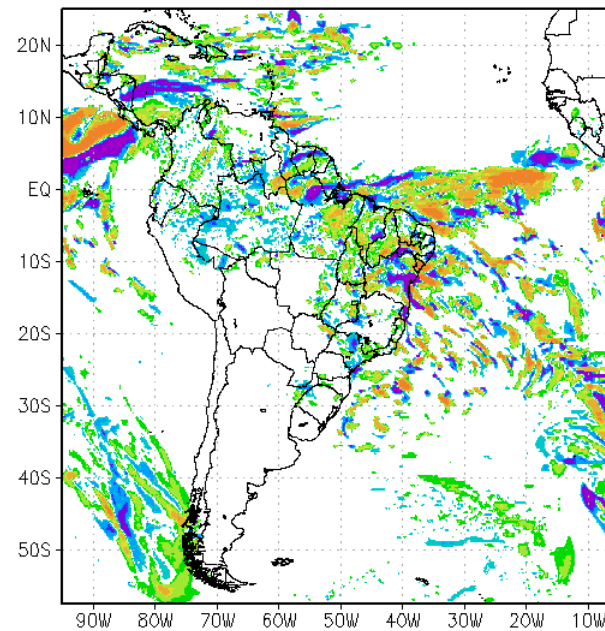
cntrl-cvisc T+72h



cntrl-cvisc T+96h



cntrl-cvisc T+120h



Difference,  
accumulated  
precip,  
mm / 24 h,  
initial condition  
27 Apr 2009



Verification not easy ...

Changes (day 5) do not appear to be random:

- Atlantic ITCZ narrower / more intense;
- Pacific ITCZ off Central America: shifted northward;
- Rainband in the Caribbean, off Nicaragua; much reduced;

...

To be continued (hopefully !!)

Two WorkEta III relevant sites:

<http://www.youtube.com/watch?v=agusq6xfefl>

<http://letras.terra.com.br/hyland-brian/18617/>

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