The advantages of the Eta model dynamics

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Deliverd by:

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8th Workshop on the Eta model numerics, characteristics, and applications

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Numerical Modeling of the Atmosphere - Dynamics

• Over the years, many numerical techniques have been with more or less success applied to modeling of atmospheric dynamics:

- Spectral, Finite-differencing, Semi-Lagrangian, Finite-elements, Finite-volumes, Spectral elements, etc.

- Each of them is focused on one or another aspect of the solution, including computational efficiency, accuracy, emulation of various local or integral properties of the fundamental equations, etc.

- Eta model, developed during 90's of the last century at the University of Belgrade, Serbia, and perfected at, at the time, NMC in the US, was developed following philosophy that:
  - Algebraic approximations should mimic integral properties of fundamental equations;
  - Numerical problems should be solved, where they are, rather than to be hidden by application of artificial filtering, diffusion, etc.

Eta dynamics: What is being done ?

- Gravity-inertia wave terms, B/E grid: forward-backward scheme that
(1) avoids the time computational mode of the leapfrog scheme, and is neutral with time steps twice leapfrog;
(2) modified to enable propagation of a height point perturbation to its nearest-neighbor height points/ suppress space computational mode;

- Split-explicit time differencing (very efficient);

- Horizontal Arakawa advection that conserves energy and C-grid enstrophy, on the B/E grid, in space differencing (Janjić 1984);

- Conservation of energy in transformations between the kinetic and potential energy, in space differencing;

- Finite-volume vertical advection of dynamic variables (**v**, T)

- Nonhydrostatic option;

- The (cut-cell) eta vertical coordinate, ensuring hydrostatically consistent calculation of the pressure gradient ("second") term of the pressure-gradient force (PGF);

These eight features make the essence of Eta dynamics!

Acting together, they:

- Increase accuracy by avoiding recognized possible errors
- Avoid “computational modes”
- Maintain integral properties
- Increase computational efficiency

Before we get into some of these:

To solve our equations we use values at grid points:

we need a horizontal grid, and a vertical grid

Horizontal

Primitive equations

Four possible square grids:

Note:

E grid is same as B but rotated 45° . Thus, often: E/B, or B/E

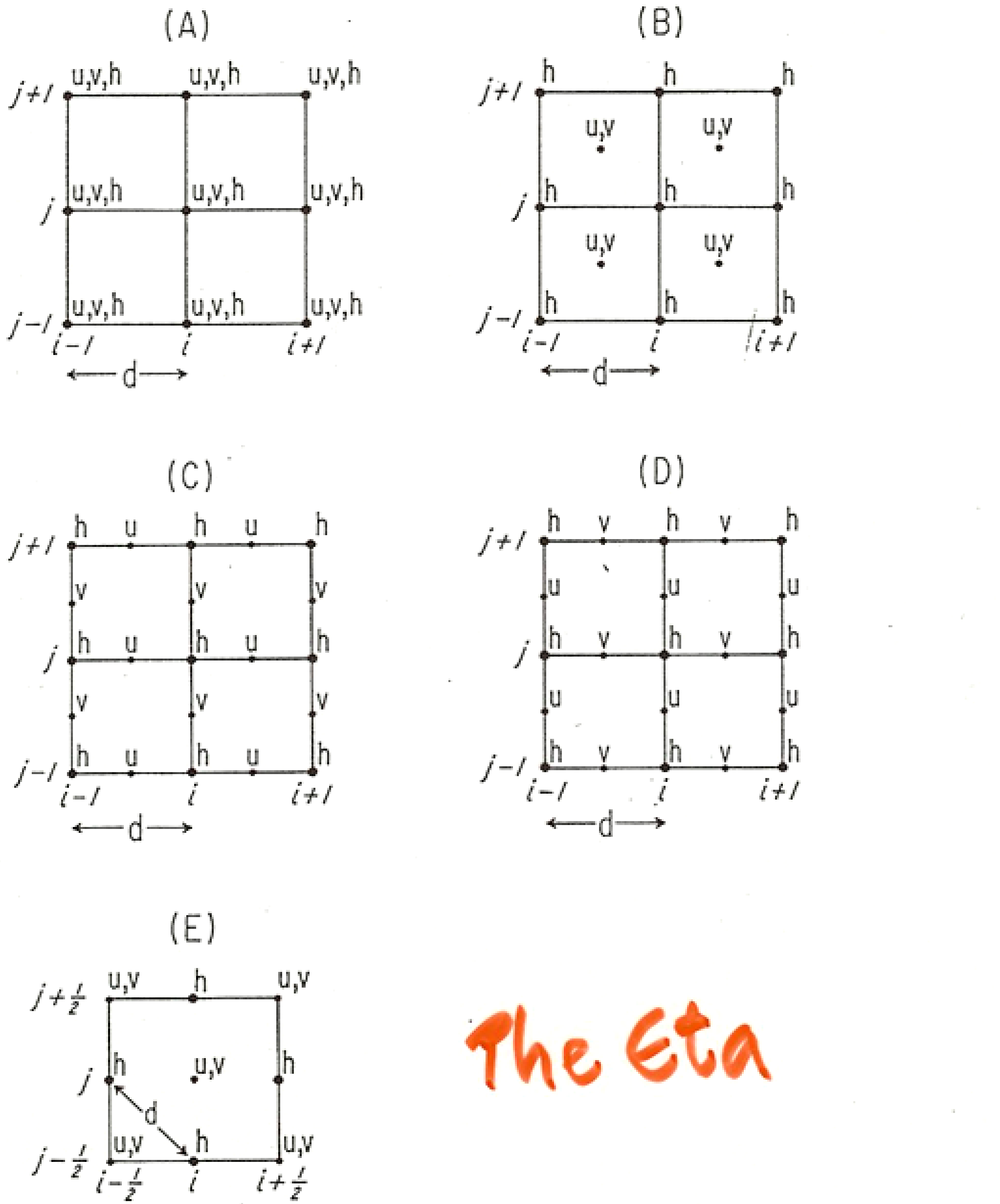


FIG. 3. Spatial distributions of the dependent variables on a square grid.

What are the values at the grid points ?

With "primitive equations", and the E grid horizontal grid consists of **v**(u,v), and T points:

v	T	**v**	T
T	**v**	T	**v**
v	T	**v**	T

Two main possibilities: values of continuous fields, taken at points, or averages over grid cells

Averages over grid cells:

Reynolds averages

This view taken in the Eta dynamics,

"Finite-volume" approach

With this approach formal, Taylor series type order of accuracy, has a questionable meaning

Perhaps the most unique and/or most beneficial:

- Horizontal Arakawa advection that conserves energy and C-grid enstrophy, on the B/E grid, in space differencing (Janjić 1984);

Early NWP and general circulation (Norman Phillips !) experience has shown that numerical models have problems in behaving quite differently – energy accumulating at small scales, with catastrophic results :(

Can one reproduce this feature of the continuous equations ?

Akio Arakawa ! (1966)

International symposium on numerical weather forecasting Oslo, March 11–16, 1963

Arakawa horizontal advection schemes

The first "general circulation" experiment:

Phillips, N. A., 1956. *The general circulation of the atmosphere: a numerical experiment*. Quart. J. Roy. Meteor. Soc., 82, 123-164.

A problem: features / energy was accumulating at small scales

Arakawa energy / enstrophy conserving schemes address

Nondivergent vorticity equation, Arakawa (1966):

$$\frac{\partial \zeta}{\partial t} + \mathbf{v} \cdot \nabla \zeta = 0; \quad \zeta = \nabla^2 \psi, \qquad (7.1)$$

where the velocity $\mathbf{v}$ is assumed to be nondivergent, that is

$$\mathbf{v} = \mathbf{k} \times \nabla \psi. \qquad (7.2)$$

Substituting this into (7.1) we obtain

$$\frac{\partial}{\partial t} \nabla^2 \psi = J\left(\nabla^2 \psi, \psi\right). \qquad (7.3)$$

From (7.13) and (7.11):

$$\overline{K}\lambda^2 = \frac{1}{2}\overline{\zeta^2} = \sum_n \lambda_n^2 K_n = \text{const}$$ as pointed out by Charney (1966):

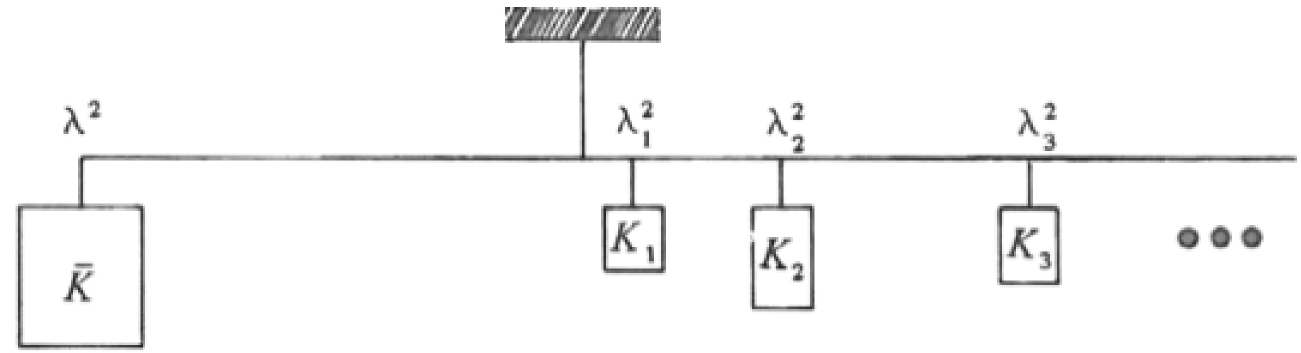


Figure 7.1 A mechanical analogy of the interchange of energy between harmonic components

Ragnar Fjørtoft
(1913-1998)

Jule Charney
(1917-1981)

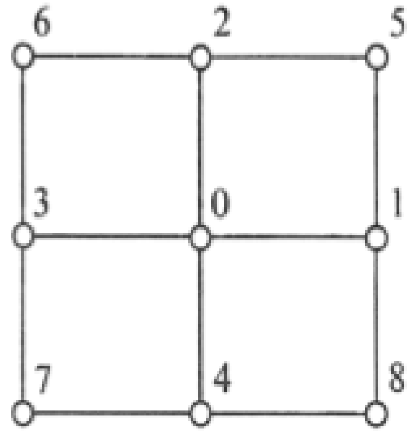
We illustrate Arakawa's method by considering how to satisfy $(7.17)_1$. In our finite difference calculation it takes the form

$$\overline{\zeta_{ij}J_{ij}(\zeta,\psi)} = \frac{1}{N}\sum_{i,j}\zeta_{ij}J_{ij}(\zeta,\psi) = 0\,, \tag{7.19}$$

$(7.17)_1$:

$$\overline{pJ(p,q)} = 0$$

$$J(p,q) = \frac{\partial p}{\partial x}\frac{\partial q}{\partial y} - \frac{\partial p}{\partial y}\frac{\partial q}{\partial x} = \frac{\partial}{\partial y}\left(q\frac{\partial p}{\partial x}\right) - \frac{\partial}{\partial x}\left(q\frac{\partial p}{\partial y}\right) = \frac{\partial}{\partial x}\left(p\frac{\partial q}{\partial y}\right) - \frac{\partial}{\partial y}\left(p\frac{\partial q}{\partial x}\right).$$



$$J^{++}(p,q) = \frac{1}{4d^2}\left[(p_1-p_3)(q_2-q_4)-(p_2-p_4)(q_1-q_3)\right], \tag{7.21a}$$

$$J^{\times+}(p,q) = \frac{1}{4d^2}\left[q_2(p_5-p_6)-q_4(p_8-p_7) - q_1(p_5-p_8)+q_3(p_6-p_7)\right], \tag{7.21b}$$

$$J^{+\times}(p,q) = \frac{1}{4d^2}\left[p_1(q_5-q_8)-p_3(q_6-q_7) - p_2(q_5-q_6)+p_4(q_8-q_7)\right]. \tag{7.21c}$$

More general :

$$J(p,q) = \alpha J^{++} + \beta J^{\times+} + \gamma J^{+\times} \tag{7.22}$$

not only do all the terms in the sum (7.19) cancel, but also all the terms in the expression for the conservation of the average kinetic energy, and the average vorticity (Arakawa, 1966; Lilly, 1965). Thus, the approximation

$$J_A \equiv \frac{1}{3}\left(J^{++} + J^{\times +} + J^{+\times}\right), \tag{7.23}$$

will conserve average vorticity, enstrophy and kinetic energy when used for the numerical solution of (7.3). This is more than sufficient for the prevention of non-linear instability. The approximation (7.23) is usually called the *Arakawa Jacobian*. Arakawa has also shown how to construct an approximation of fourth order accuracy to the Jacobian, conserving these three quantities.

Arakawa vorticity equation scheme transformed to the C-grid:

Arakawa A. and V. R. Lamb, 1977: Computational design of the basic dynamical processes of the UCLA general circulation model. *Methods in Computational Physics,* J. Chang, Ed., Academic Press, 174–264. ("The Green Book")

The C-grid Arakawa scheme transformed to the B/E-grid:

Janjić, Z. I., 1984: Nonlinear advection schemes and energy cascade on semi-staggered grids. *Mon. Wea. Rev.*, **112**, 1234-1245.

Rančić, M., 1988: Fourth-order horizontal advection schemes on the semi-staggered grid. *Mon. Wea. Rev.*, **116**, 1274-1288.

From ECMWF Seminar 1983:

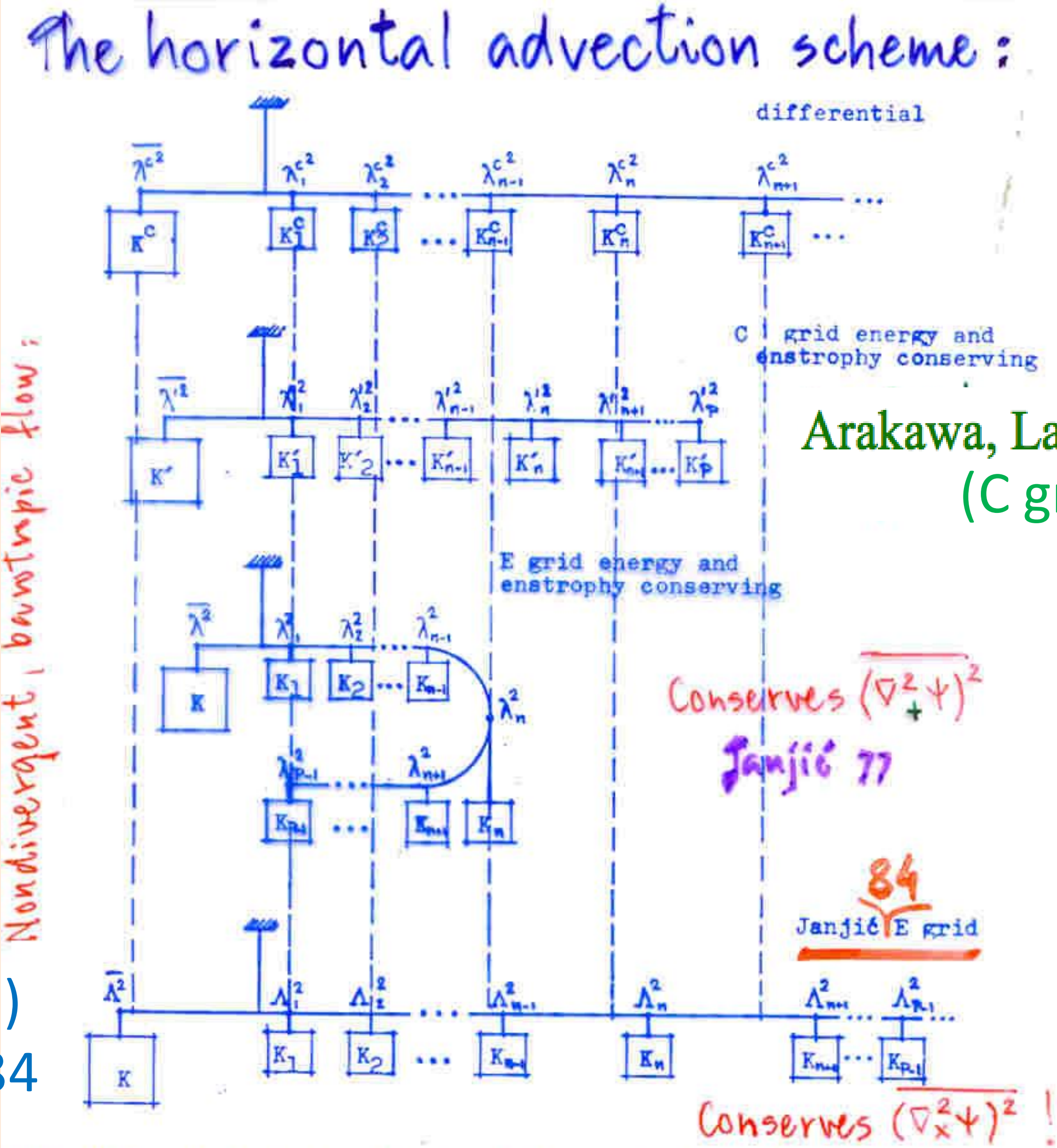


Fig. 3.12. Mechanical analogies of the constraints imposed on the non-linear energy cascade in the continuous case, in the case of the C-grid energy and enstrophy conserving scheme, in the case of the E-grid energy and enstrophy conserving scheme, and in the case of the scheme due to Janjić (1984).

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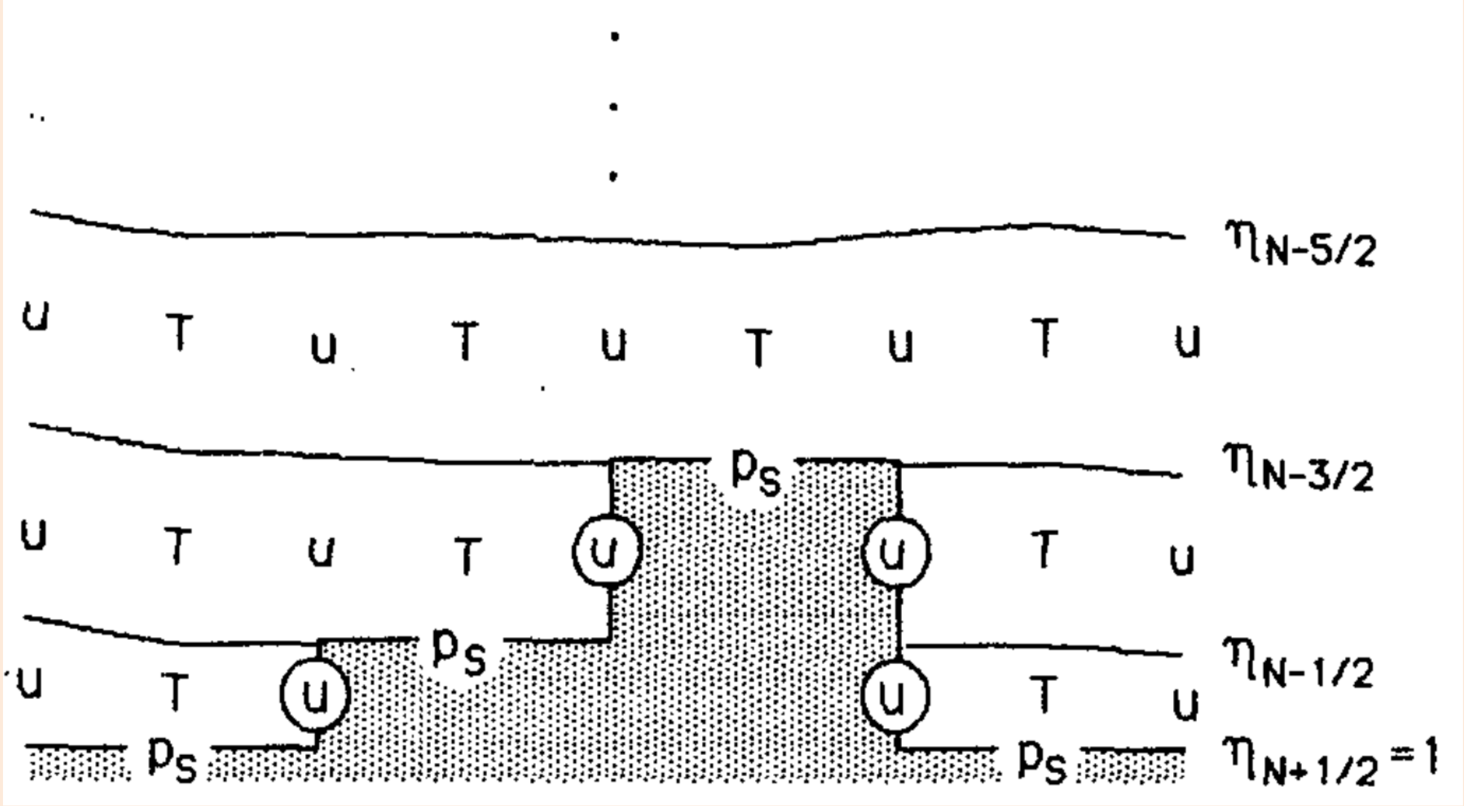
Arakawa, Lamb (1977) (C grid)

(B/E grid) Janjić 1984

Unique feature of Eta model is the cut cell treatment of terrain

- We start with description of the step coordinate model, that lead to cut cell formulation

--> The Eta model

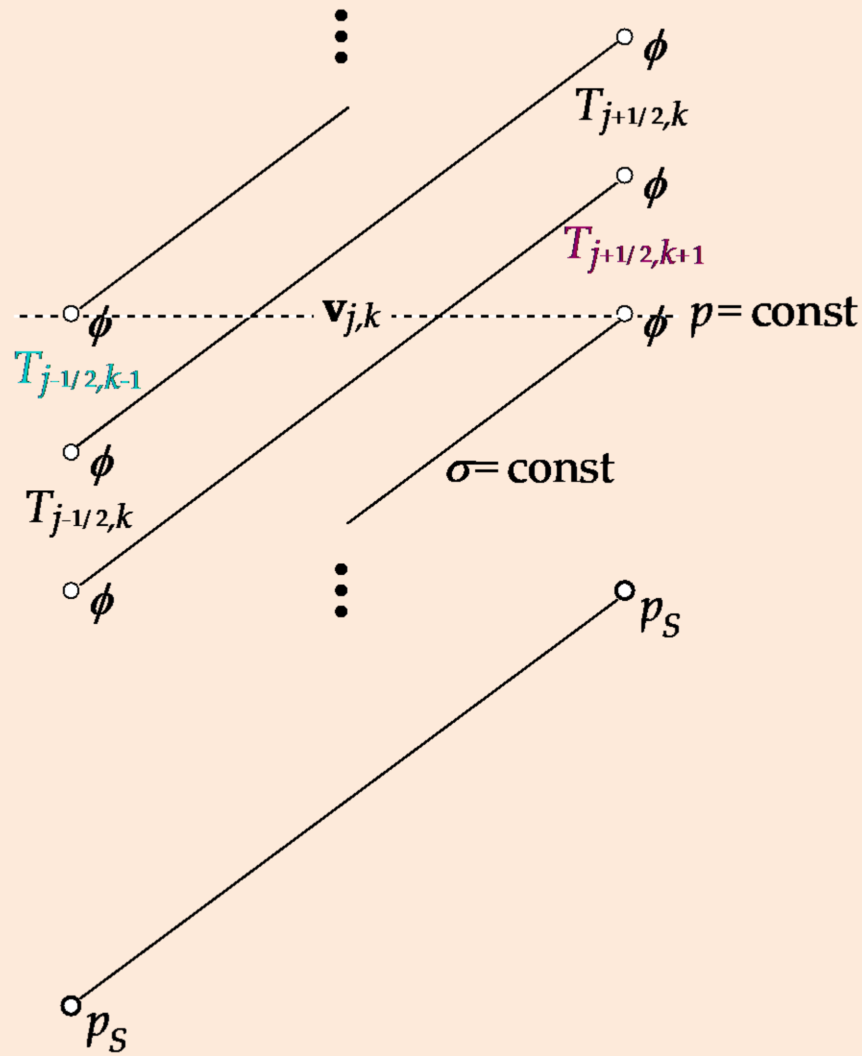


The eta vertical coordinate

Terrain-following coordinates: pressure gradient force has problems !

Continuous case:
PGF should depend on,
and only on,
variables from the ground
up to the p=const surface:

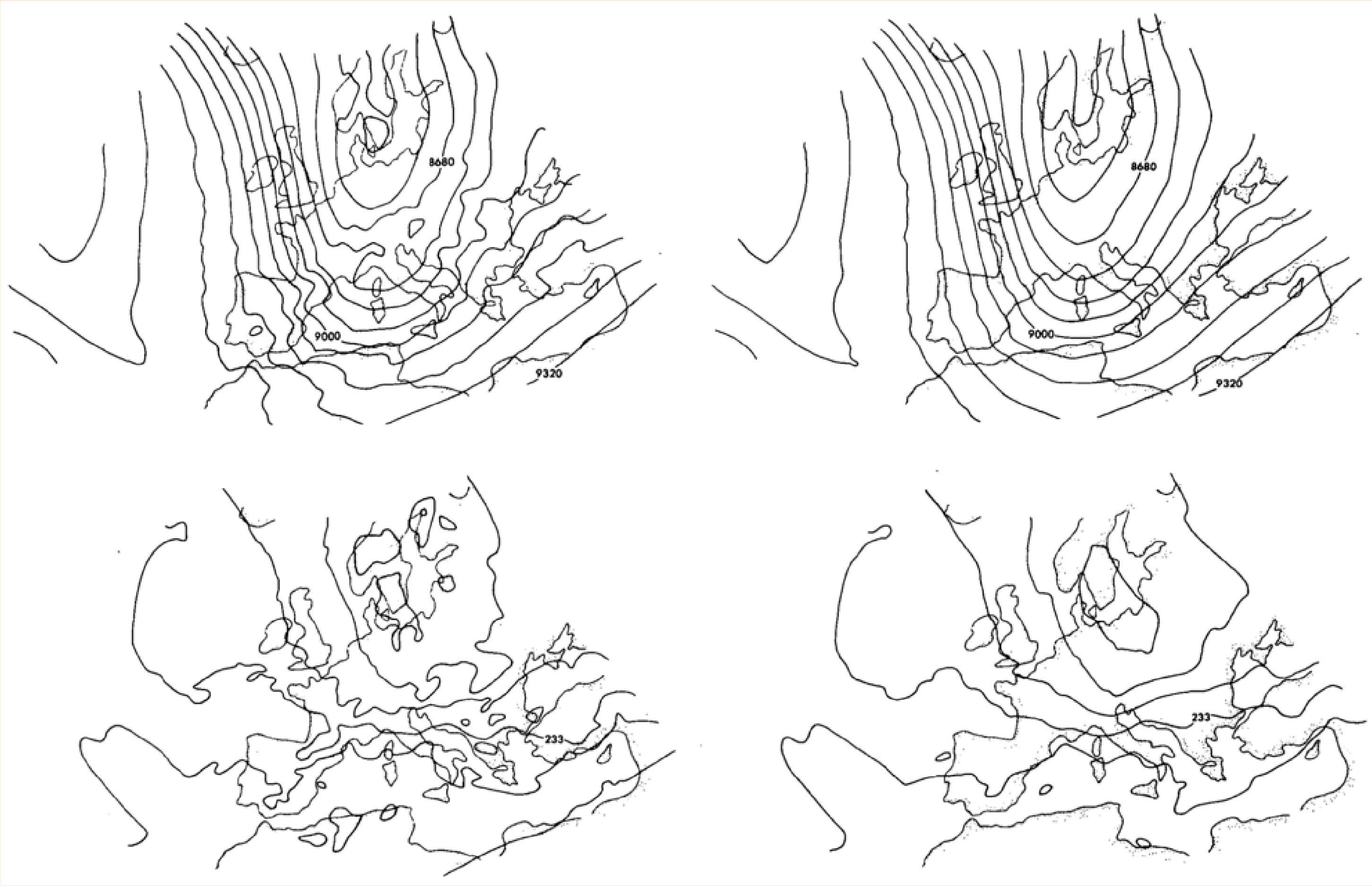
$$-\nabla_p \phi = -\nabla_\sigma \phi - \frac{RT}{p} \nabla_\sigma p$$



Difference of two large terms always will create the error!

No remedy using locally increased resolution!!!

300 hPa geopotential heights (above) and temperatures (below) in a 48-h simulation using the sigma system (left) and using the eta system (right). Contour intervals are 80 m for geopotential heights, and 2.5 K for temperature. From Mesinger et al. (1988)



Ability to circumvent problem related to pressure gradient error around high terrain was instrumental for selecting Eta model for the main regional model of the US Weather Service in 1993.

Eta became primary regional operational model at U.S. National Meteorological Center as of March 1993. At INPE, 1996. . . . . .

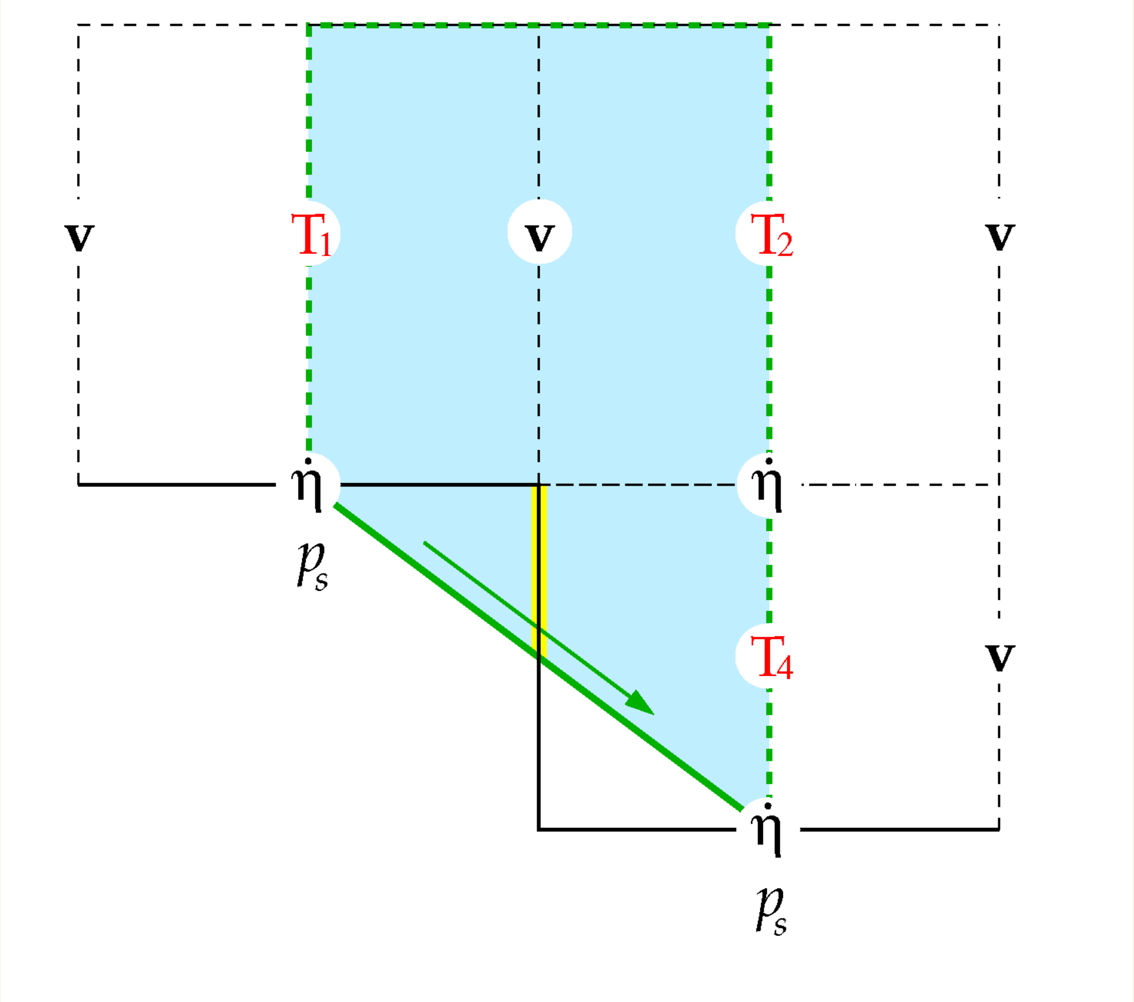
However,

- Experimental 10-km Eta did poorly a windstorm in the lee of Wasatch mountain, while a sigma system MM5 did well,
- Gallus and Klemp (2000) published experiments on flow over a bell-shaped topography. Gallus and Rančić (1994) eta coordinate model failed to simulate downstream flow, instead had the flow in the lee separate off the top of the topography

Gallus and Klemp ascribed the problem to the existence of step corners of the step topography Eta, therefore:

The sloping steps (a simple cut-cell scheme), vertical grid:

The central **v** box exchanges momentum, on its right side, with **v** boxes of two layers, and T_1 box undergoes horizontal advection to T_2 and vertical (slantwise) advection to T_4



From Mesinger, Jovic (NCEP Office Note 2002):

Slope adjustment scheme

Slopes can be adjusted, but no new maxima or minima must be created. This is the first iteration. If we are not next to a minimum or maximum, we can go only half of the smaller of two sides

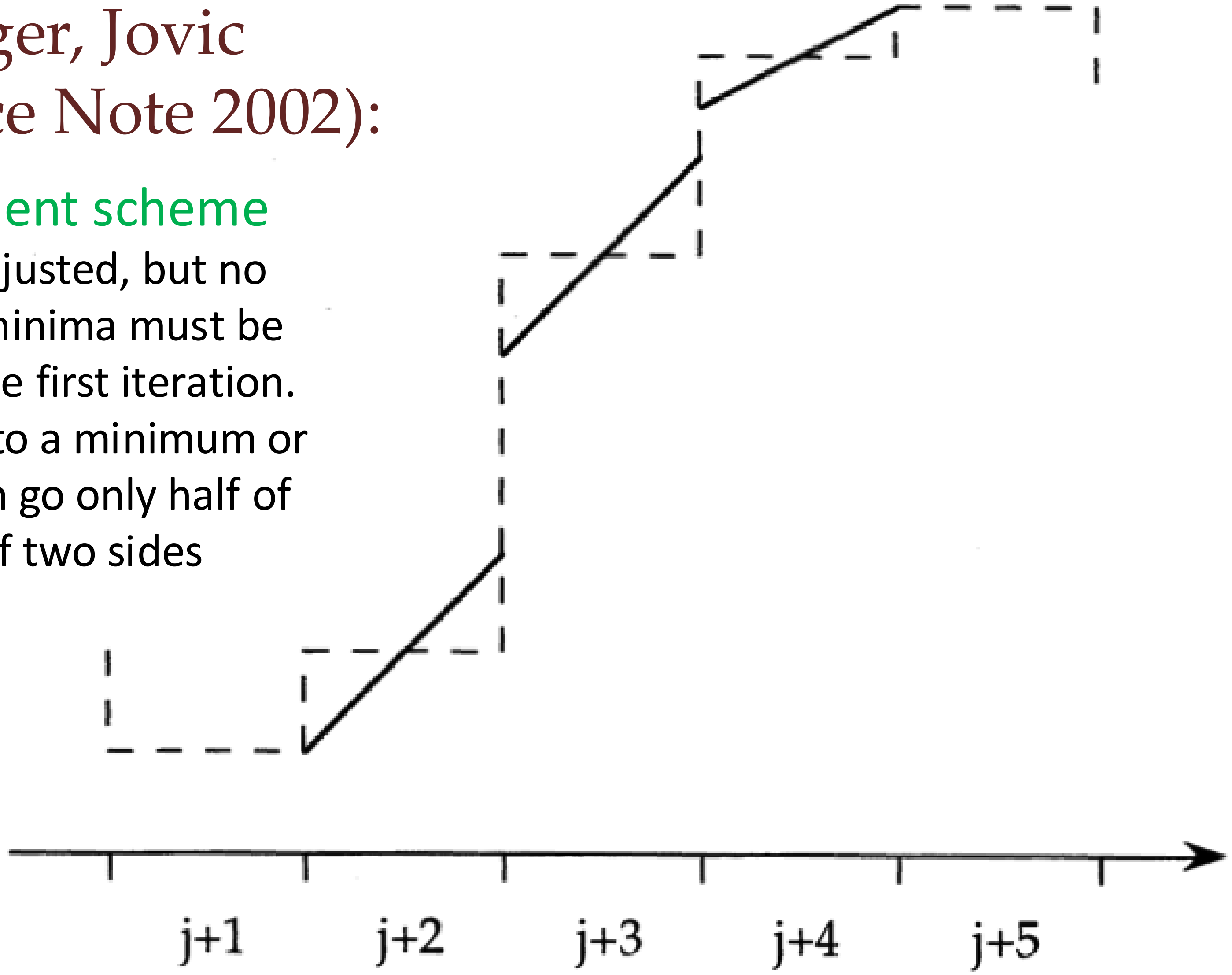
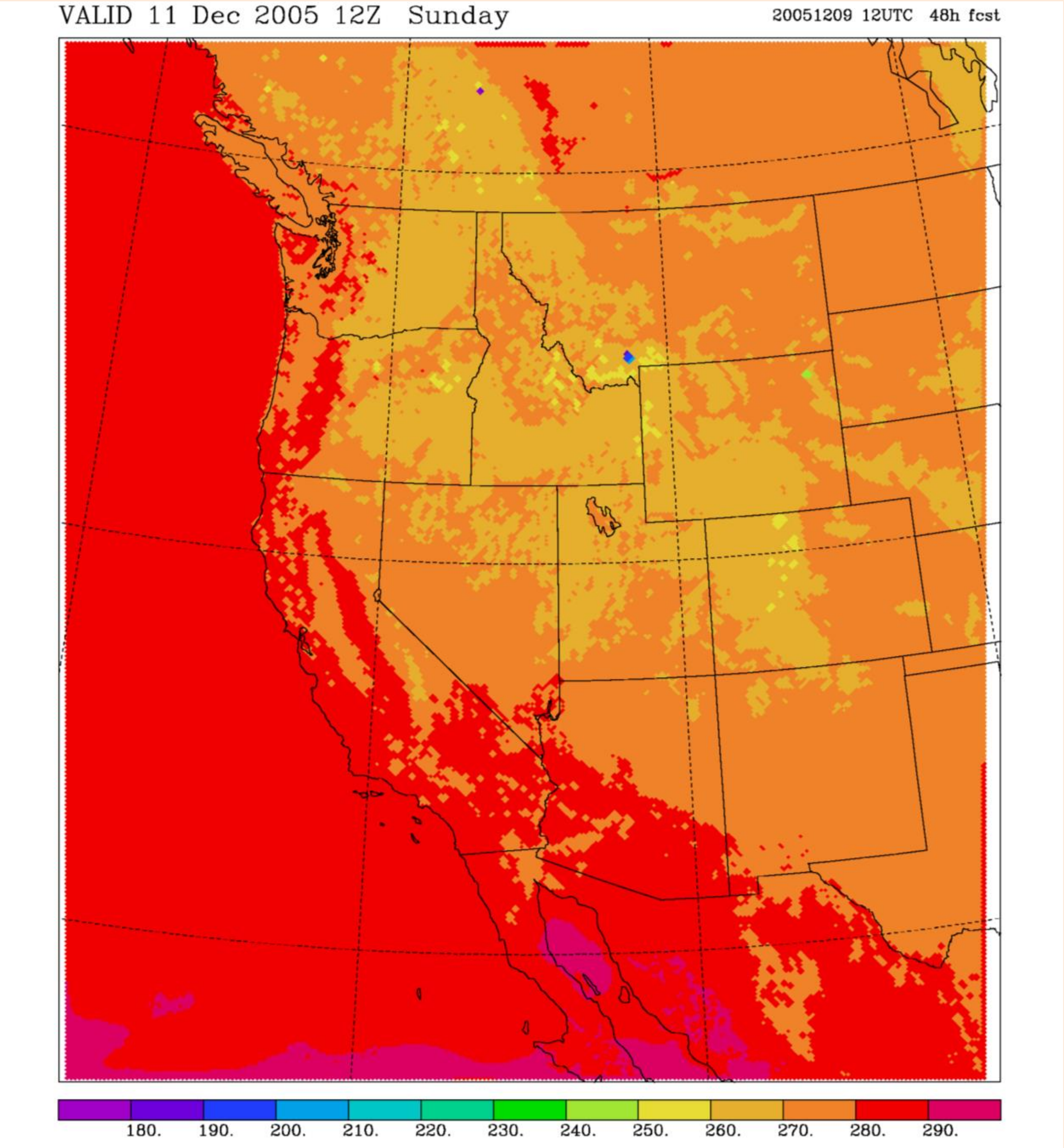


Figure 1. An example of the Eta iterative slope adjustment algorithm. The initial distribution is illustrated by the dashed line, with slopes in all five zones shown equal to zero. Slopes resulting from the first iteration are shown by the solid lines. See text for additional detail.

When this was coded and tested, 48-h lowest T boxes map:



Situation was mostly improved but not yet fully solved :

- Which motivated work on improvement of vertical advection scheme.

Suspect: slantwise T advection:

standard "Lorenz-Arakawa" centered vertical advection scheme (Arakawa and Lamb 1977)

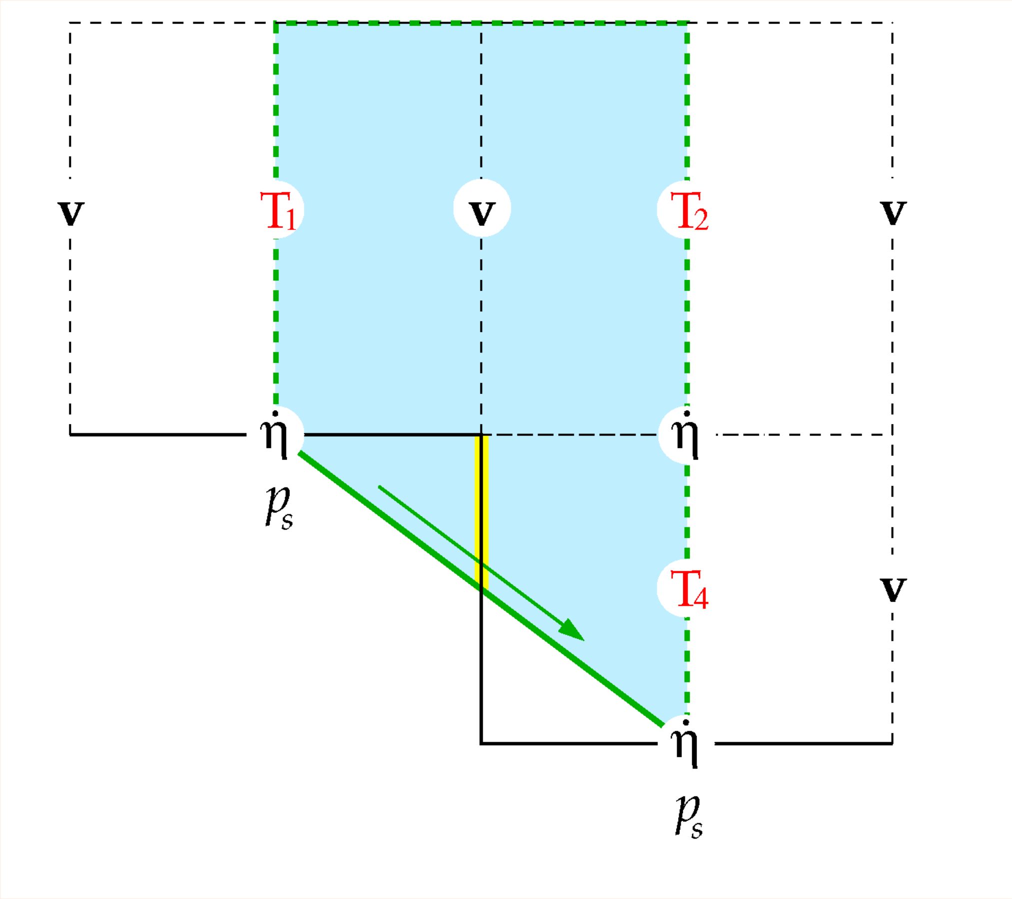
$$\frac{\partial T}{\partial t} = \cdots - \dot{\eta}\,\overline{\frac{\partial T}{\partial \eta}}^{\eta} \qquad (2)$$

It allows a false vertical advection from below ground !!

If a temperature inversion were to develop at the bottom of a basin, with a persistent upward motion, then the vertical advection contribution from the interface between the lowest T cell and the one above it would cool both cells, but for the lower of them would be the only contribution, thus tending to increase the inversion, amplifying its cooling, feeding on itself !!!

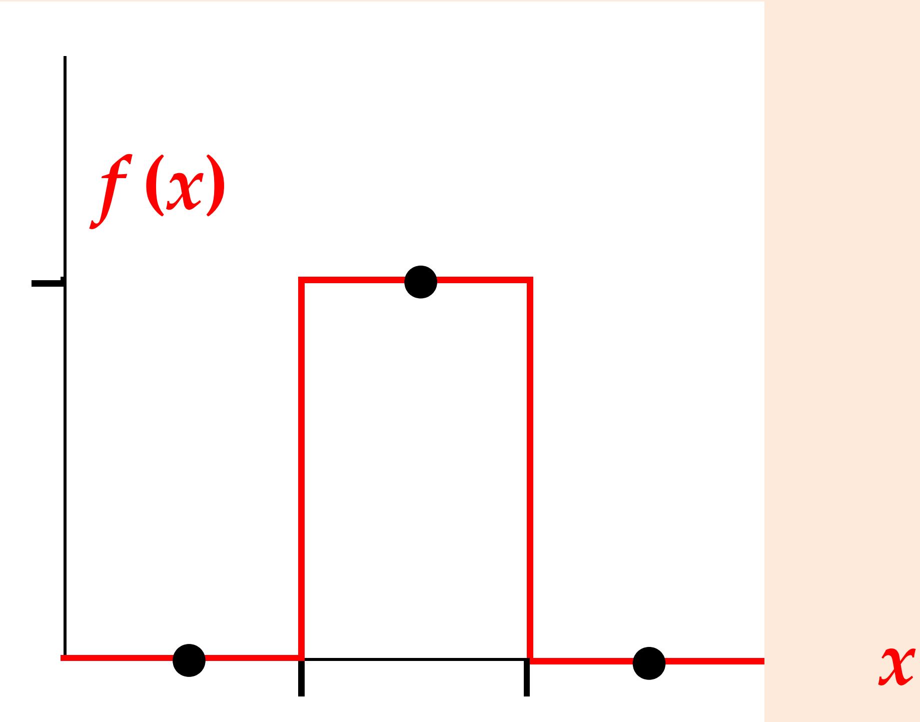
In addition, this advection into the lowest cell, is physicaly wrong, no advection should exist into the lowest cell from below ground !!

But with the finite-volume approach, with **v** constant inside the bluish **v** cell, as well as the T_1 and T_4 inside their cells, we can calculate how much air is crossing the yellow line and replace the wrong slantwise advection with correct T changes !!!



Piecewise linear (finite-volume) advection scheme used

Consider advection of a top hat (or step) function, e.g.:



Creation of false maxima or minima using centered schemes !

Advection of moisture :(

Minmod limiter:

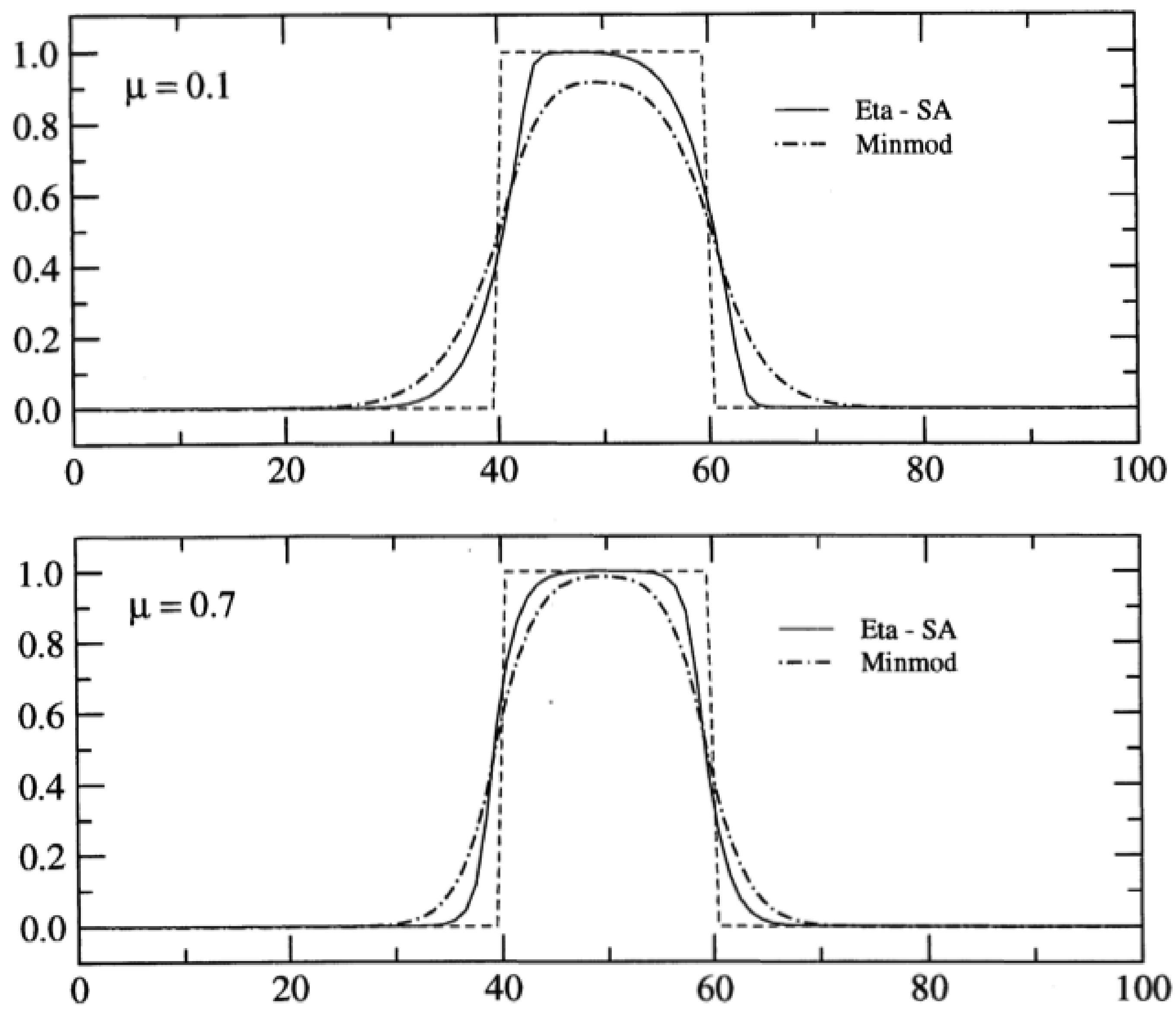
$C = C(r),$

$r \equiv \dfrac{q_j - q_{j-1}}{q_{j+1} - q_j}$

$$C(r) = \max\langle 0, \min(1, r)\rangle$$

defines slope to be that of the smaller, in absolute value, of the two boundary values of $\Delta q/\Delta x$, unless q_j is is an extremum in which case the slope is zero

(Durran 1999, and also 2010, Fig. 5.16.)



After two translations of the true solution across the domain

Figure 4. Same as Fig. 2, except for the Eta slope-adjustment scheme results (SA, solid line) compared against those using the minmod slope limiter (dot-dashed line). See text for definitions of schemes.

Monotonized-centered limiter:

$$C(r) = \max\left\langle 0, \min(2r, \frac{1+r}{2}, 2)\right\rangle$$

(also van Leer 1977) algebraic average of the two boundary slopes (same as using a centered scheme), unless this violates the monotonicity condition in which case they are reduced to the extent required. If however q_j is an extremum the slope is again set to zero

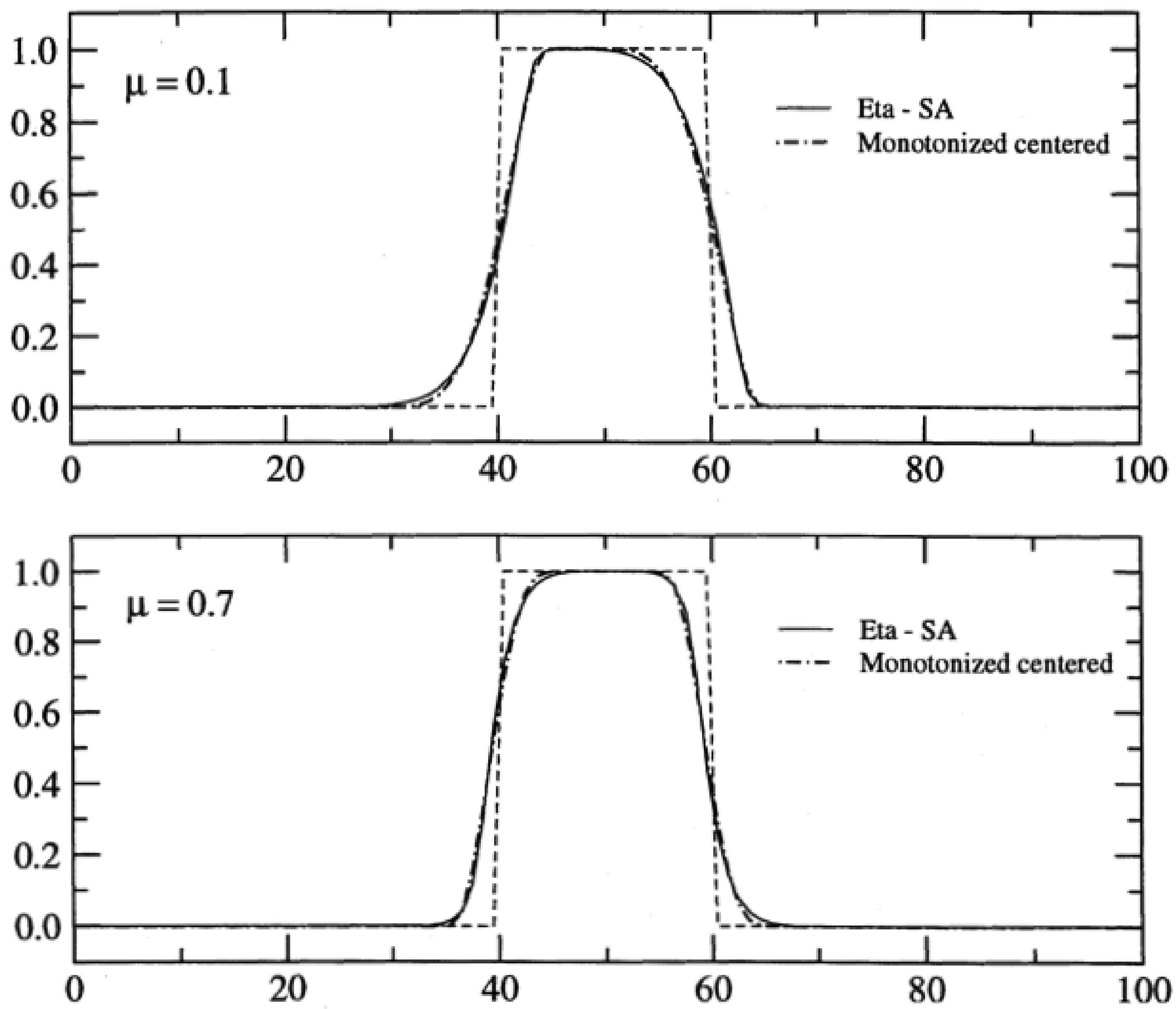


Figure 6. Same as Fig. 2, except for the Eta slope-adjustment scheme results (SA, solid line) compared against those using the monotonized centered slope limiter (dot-dashed line). See text for definitions of schemes.

Takacs' 3rd order scheme

(3rd order when its parameter α is a given function of μ)

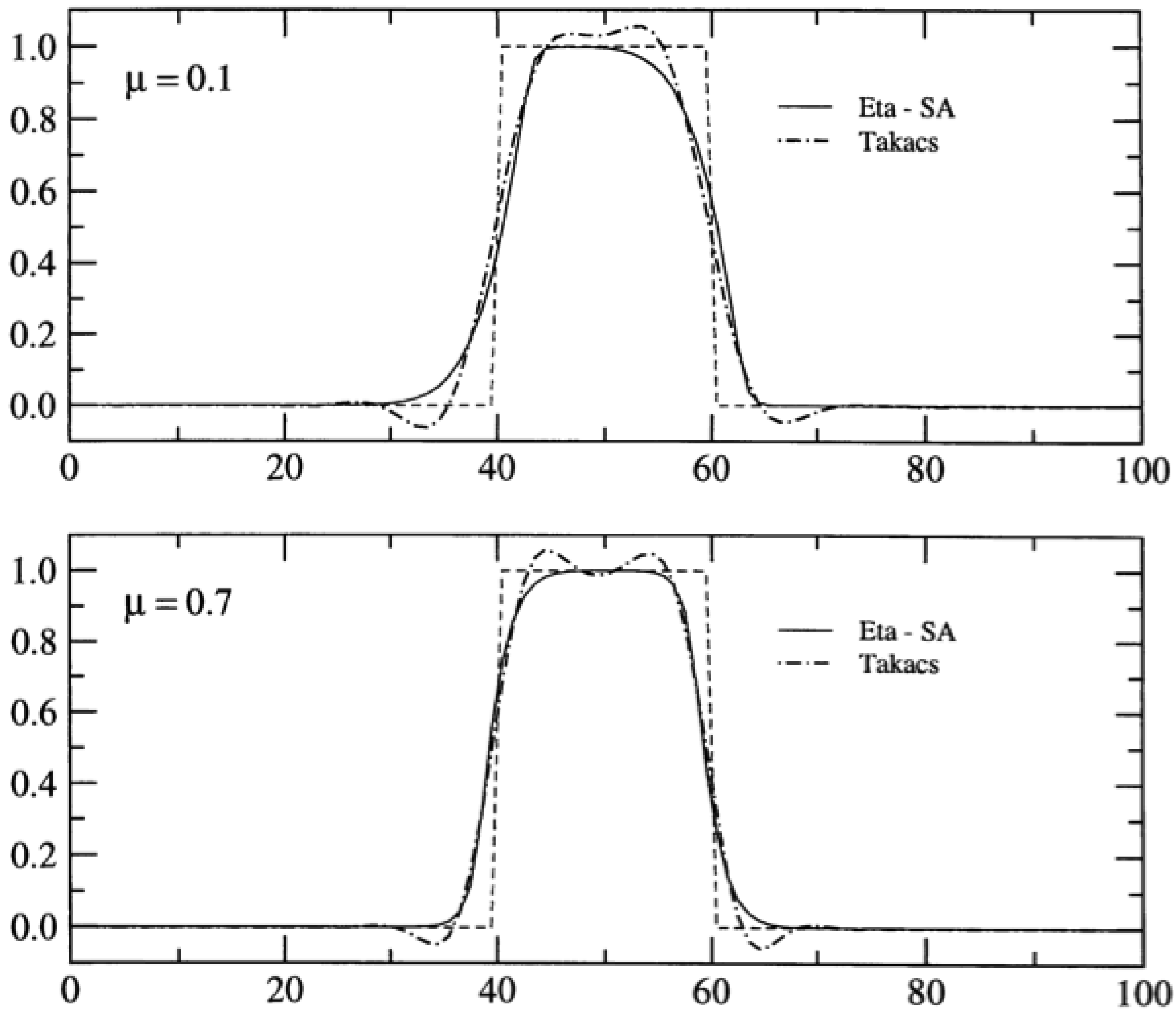
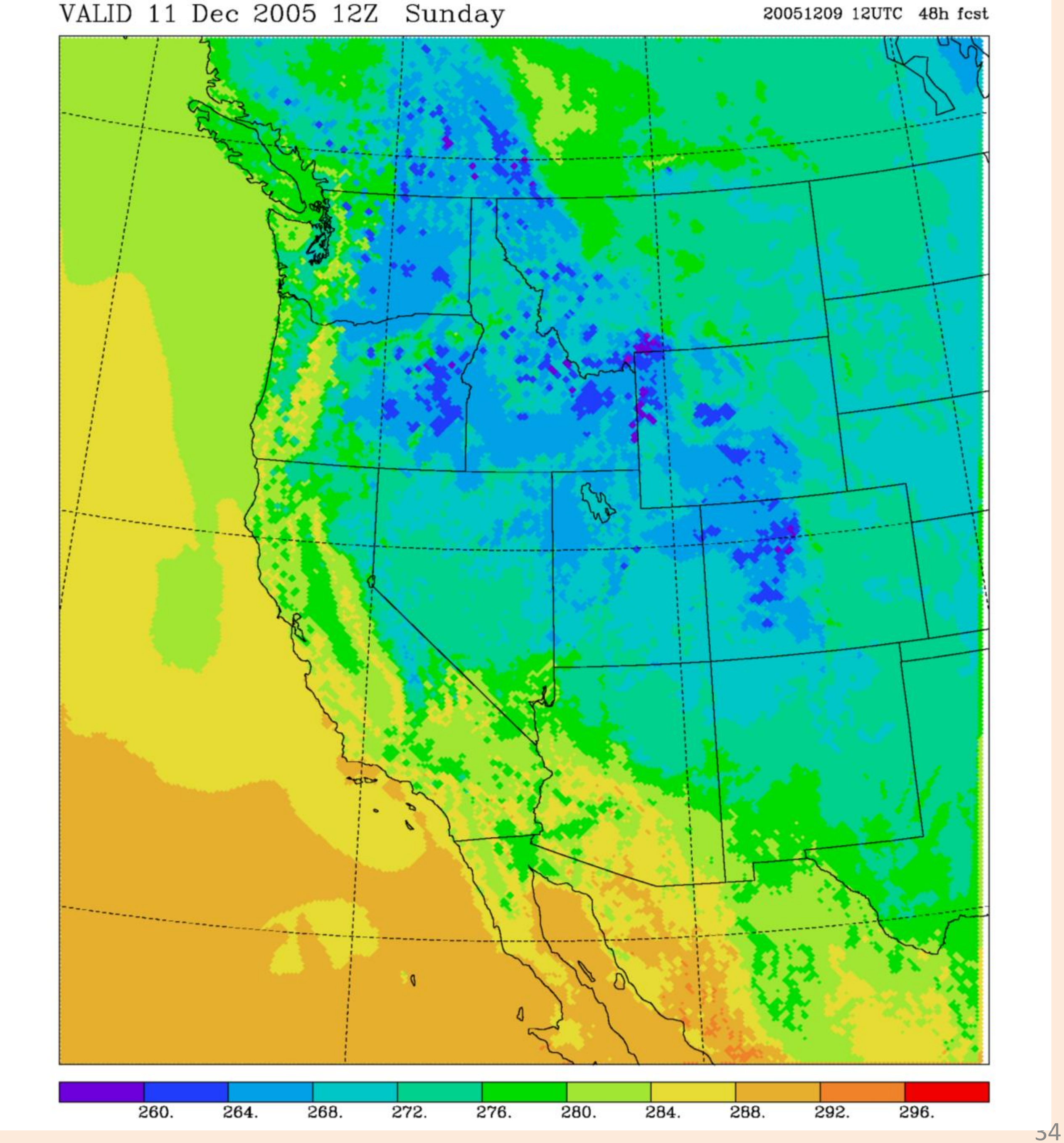


Figure 9. Same as Fig. 2, except for the Eta slope-adjustment scheme results (SA, solid line) compared against those using the Takacs (1985) third-order "minimized dissipation and dispersion errors" scheme (dot-dashed line). See text for definitions of schemes.

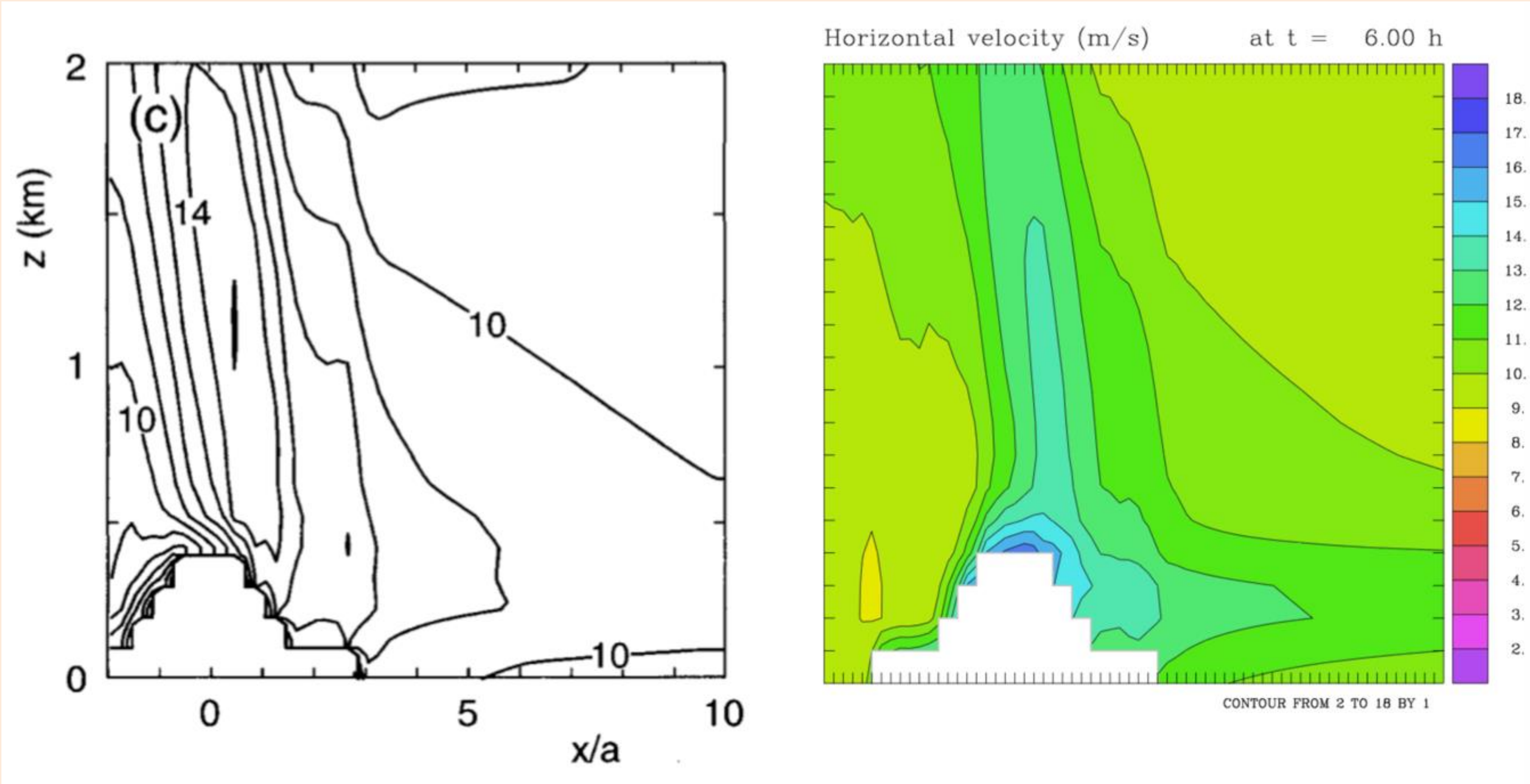
A still more ambitious scheme:

Rančić M., 1992: Semi-Lagrangian piecewise biparabolic scheme for two-dimensional horizontal advection of a passive scalar. *Mon. Wea. Rev.*, **120**, 1394-1406.

With finite difference scheme of slide (40) replaced by the Lagrangean slantwise advection, and the van Leer type SA scheme for vertical advections of all prognostic variables, 48-h lowest T values now



And the Gallus-Klemp problem:



Simulation of the Gallus-Klemp experiment with the Eta code, plot (c) of Fig. 6 of Gallus and Klemp (2000), left, using the sloping steps Eta code allowing for velocities at slopes in the horizontal diffusion scheme, right. From Mesinger and Veljovic (Meteor Atmos Phys, 2017).

Several test results:

Accuracy

of a model, ran using real data IC

Issues:

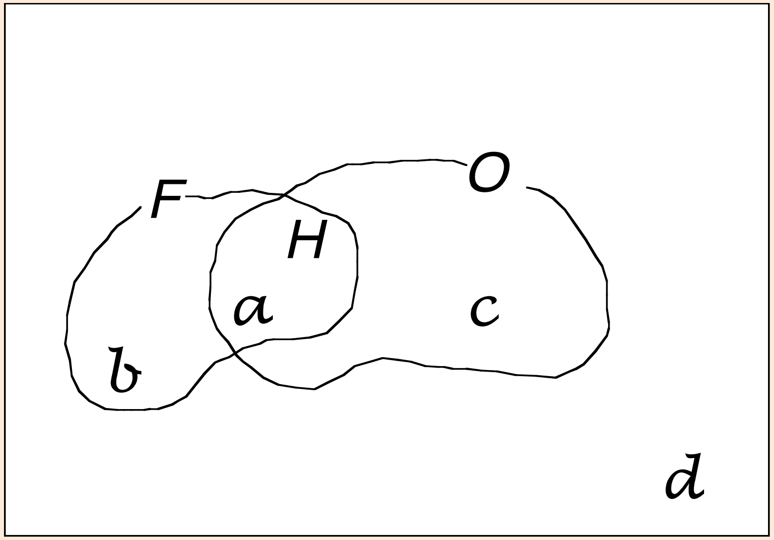
Atmosphere is chaotic,

Results depend on data assimilation system / the IC

Impacts of both are avoided if we drive our limited area “test model” by ICs and LBCs of an ensemble of a global model

Accuracy of the jet stream position . . . .

Forecast, Hits, and Observed (*F*, *H*, *O*) area,
or number of model grid boxes:



Many verification scores.

One:

$$ETS = \frac{H - E(H)}{F + O - H - E(H)}$$

"Equitable Threat Score"

or, Gilbert (1884 !) Skill Score

Bias = *F* / *O*

ETS corrected (adjusted) for bias: ETS_a:

Mesinger F, 2008: Bias adjusted precipitation threat scores. *Adv. Geosci.*, **16**, 137-143 (open access).

ECMWF once a week runs a 51 - member ensemble forecast 32 days ahead

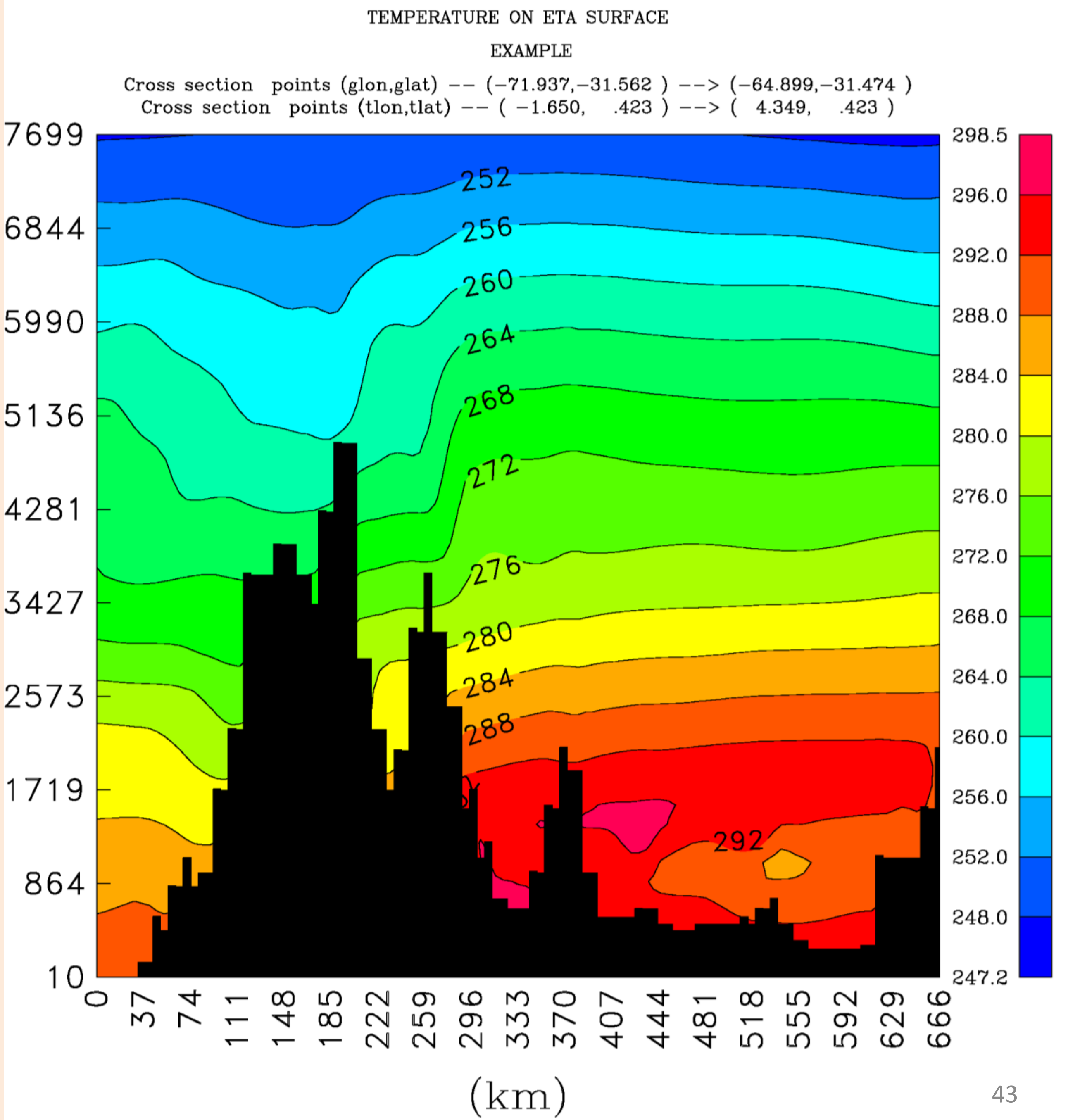
Mesinger F, Chou SC, Gomes J, Jovic D, Bastos P, Bustamante JF, Lazic L, Lyra AA, Morelli S, Ristic I, Veljovic K (2012) An upgraded version of the Eta model. Meteorol Atmos Phys **116**, 63–79. doi:10.1007/s00703-012-0182-z

Mesinger, F, Veljovic K (2017) Eta vs. sigma: Review of past results, Gallus-Klemp test, and large-scale wind skill in ensemble experiments. Meteorol Atmos Phys, **129**, 573-593, doi:10.1007/s00703-016-0496-3

8 km horizontal resolution,

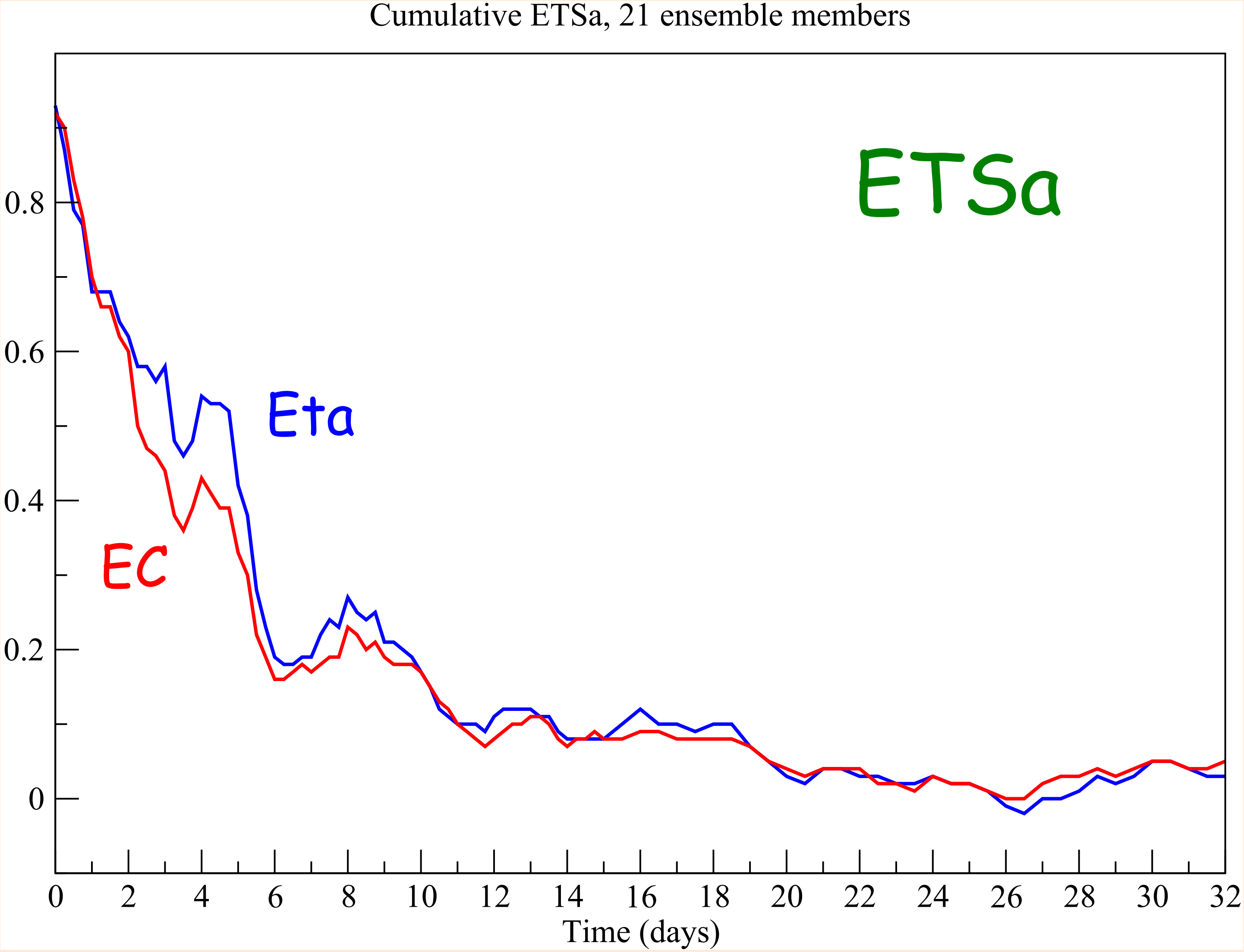
W/E profile at the latitude of about the highest elevation of the Andes

NCAR graphics, no cell values smoothing



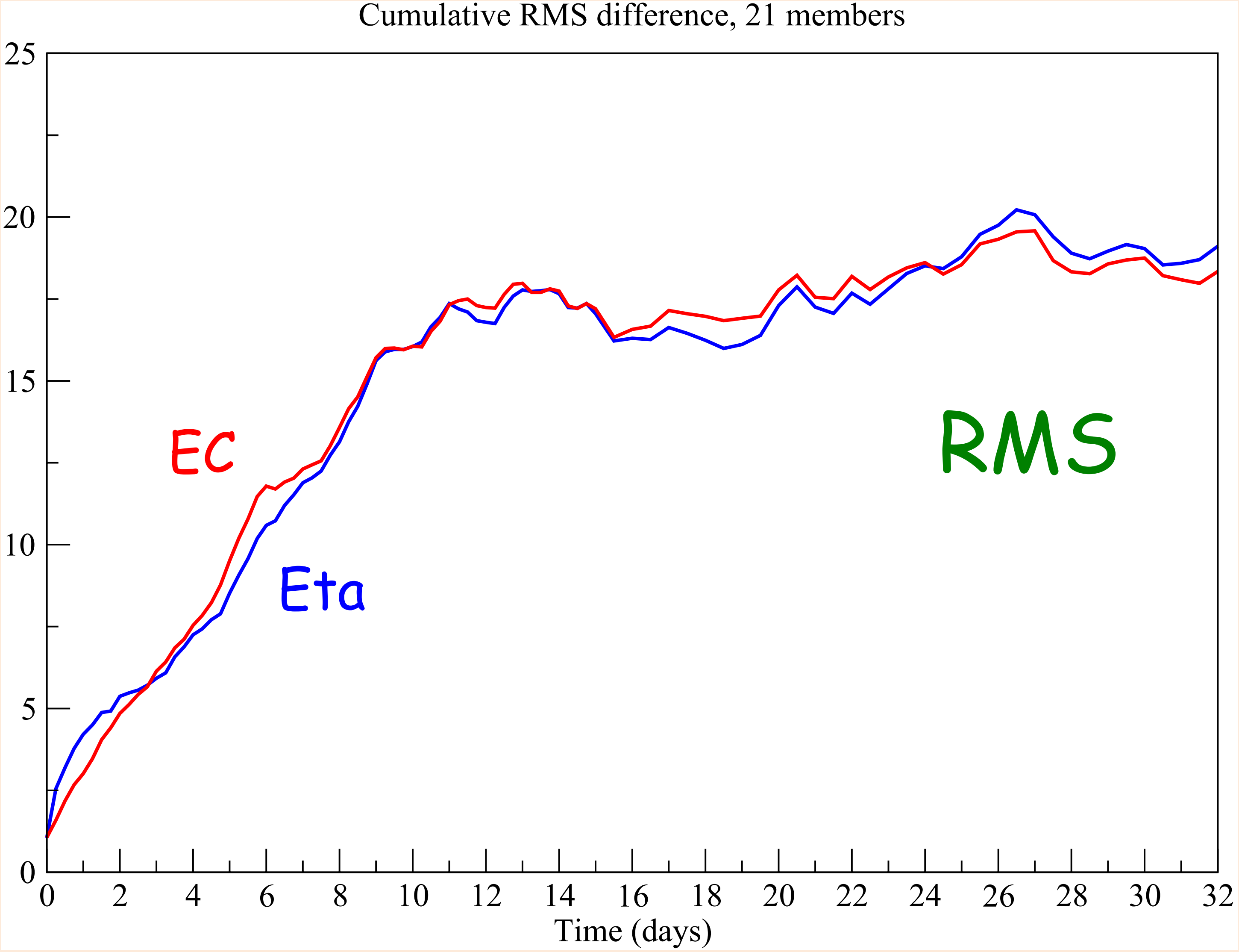
Verification results

21 ensemble members



Bias adjusted ETS scores of wind speeds > 45 m s^{-1}, at 250 hPa, with respect to ECMWF analyses

ETSa: More is better !



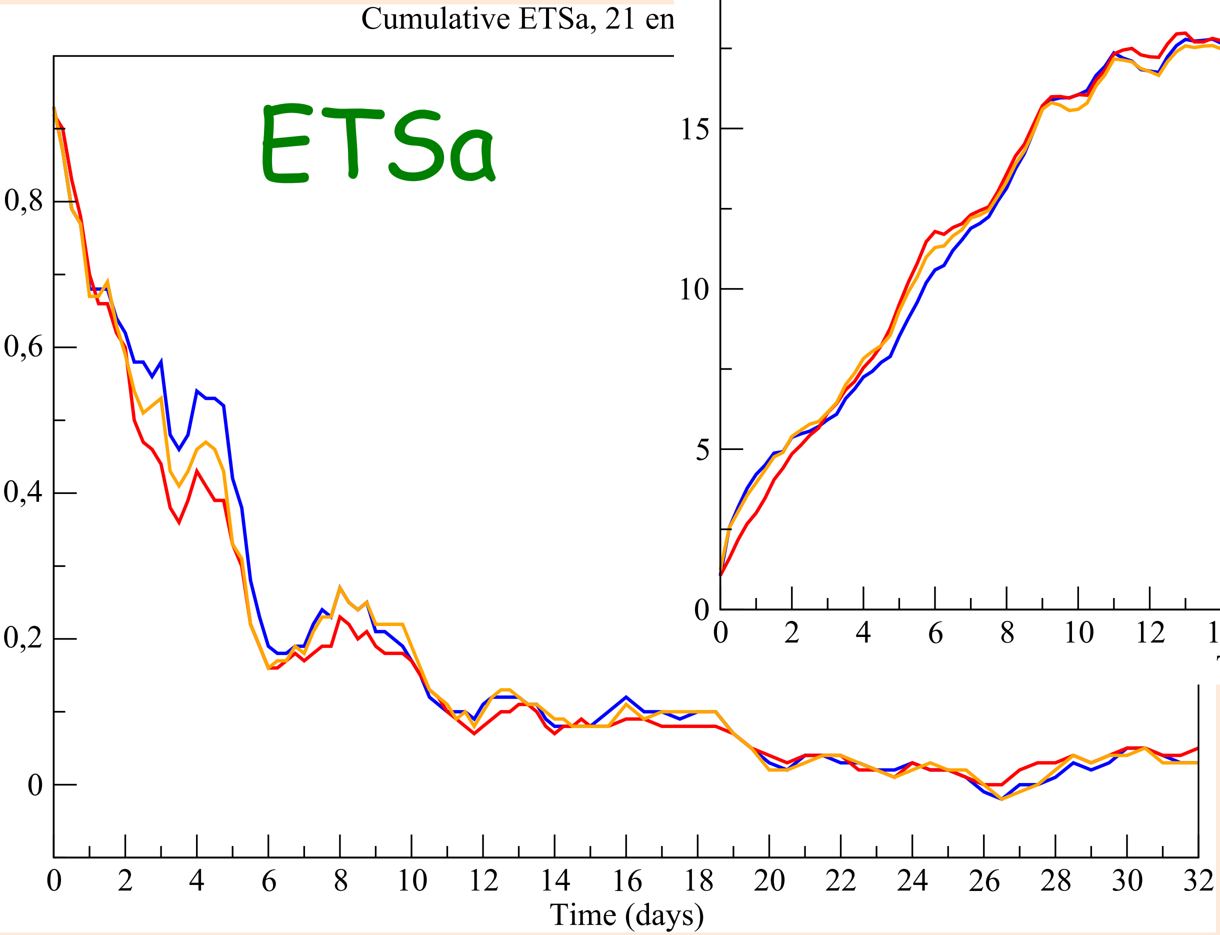
RMS wind difference of 250 hPa winds, with respect to ECMWF analyses

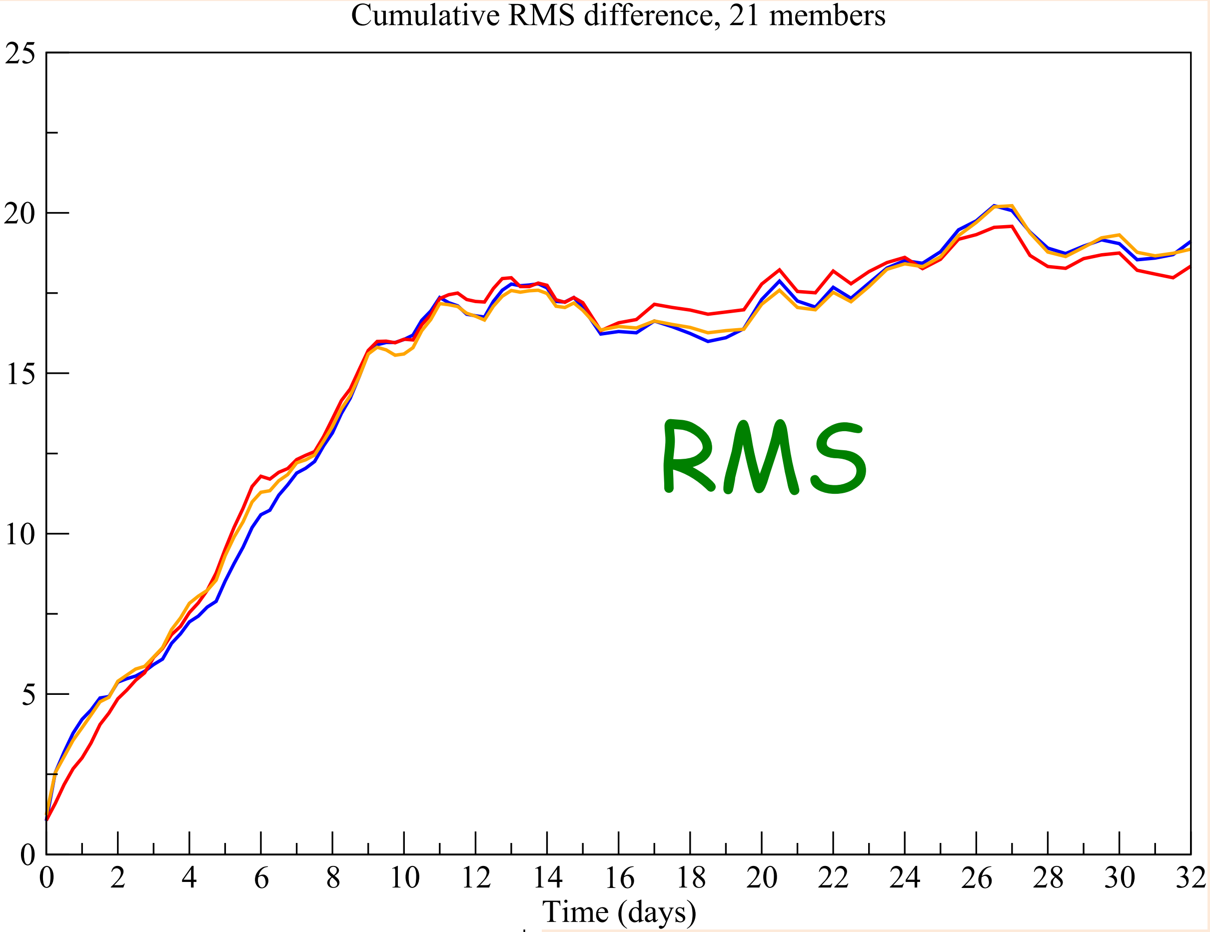
RMS: Less is better !

What ingredient of the Eta is responsible for the advantage in scores ?

(It is not resolution, the first 10 days resolution of two models was about the same)

21 members ran using Eta/sigma:

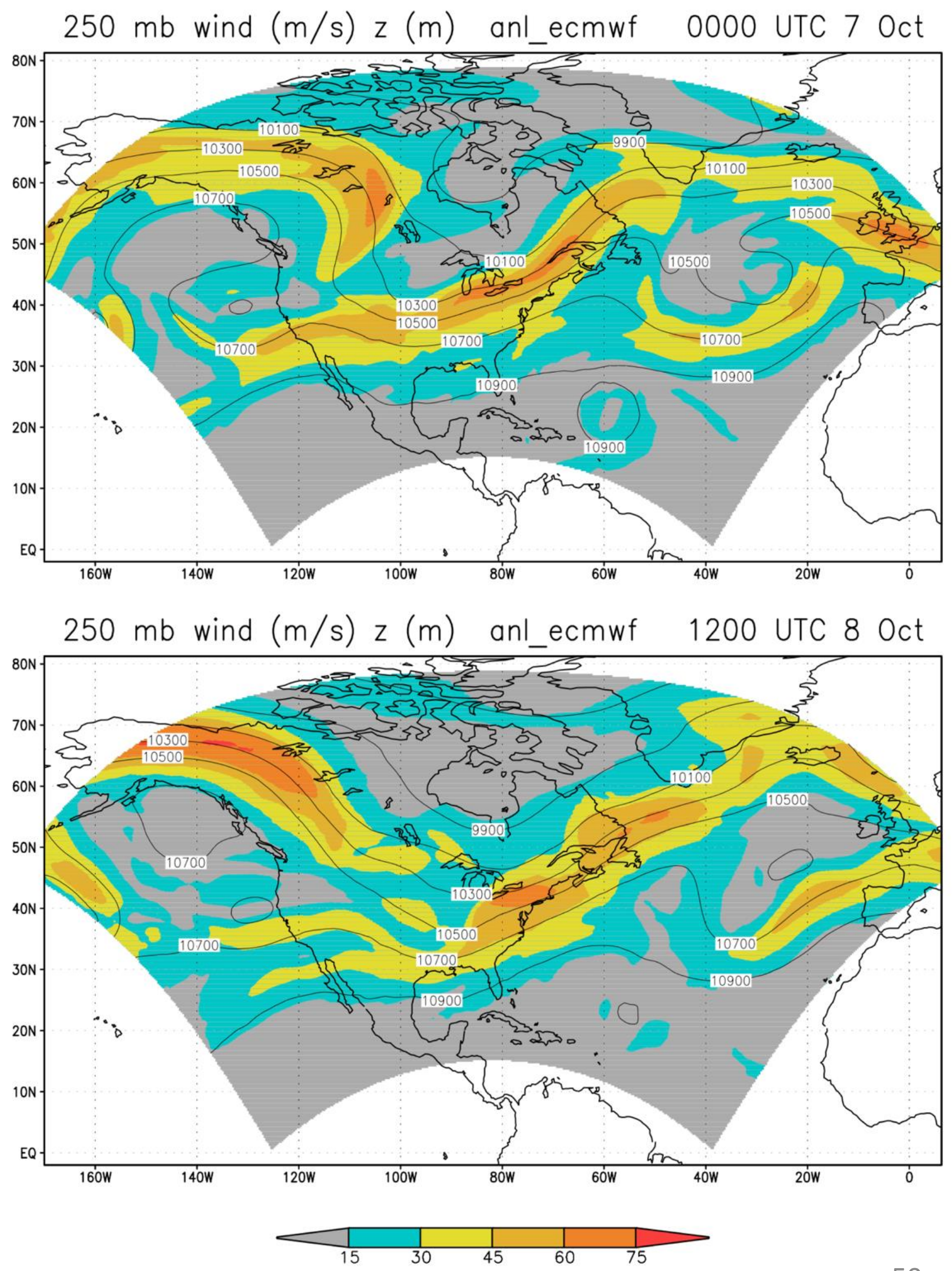




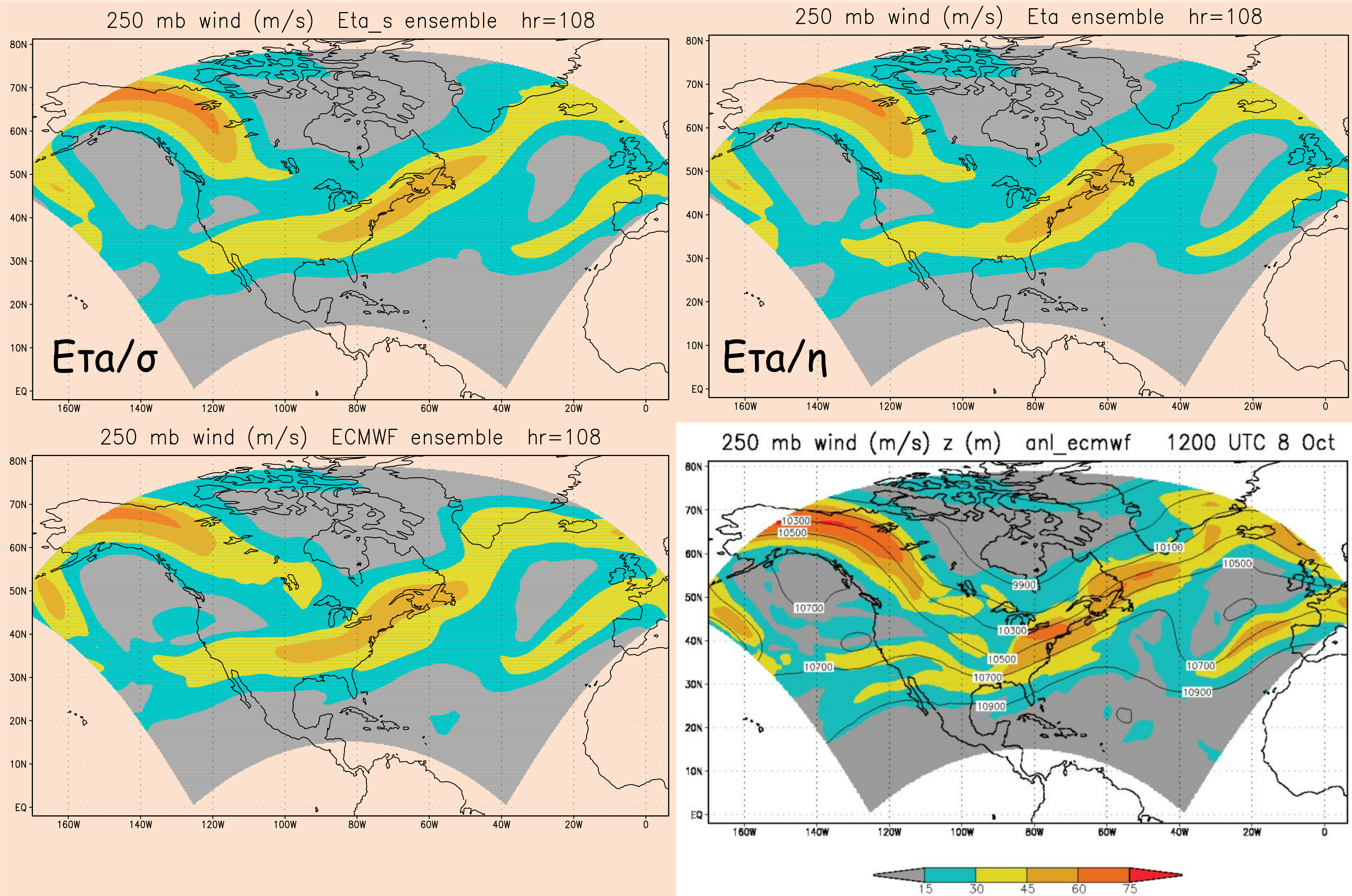
What was going on
at about day
2-6 time ?

What was going on at about day 2-6 time ?

The plot times correspond to day 3.0, and 4.5, respectively, of the plots of the two preceding slides



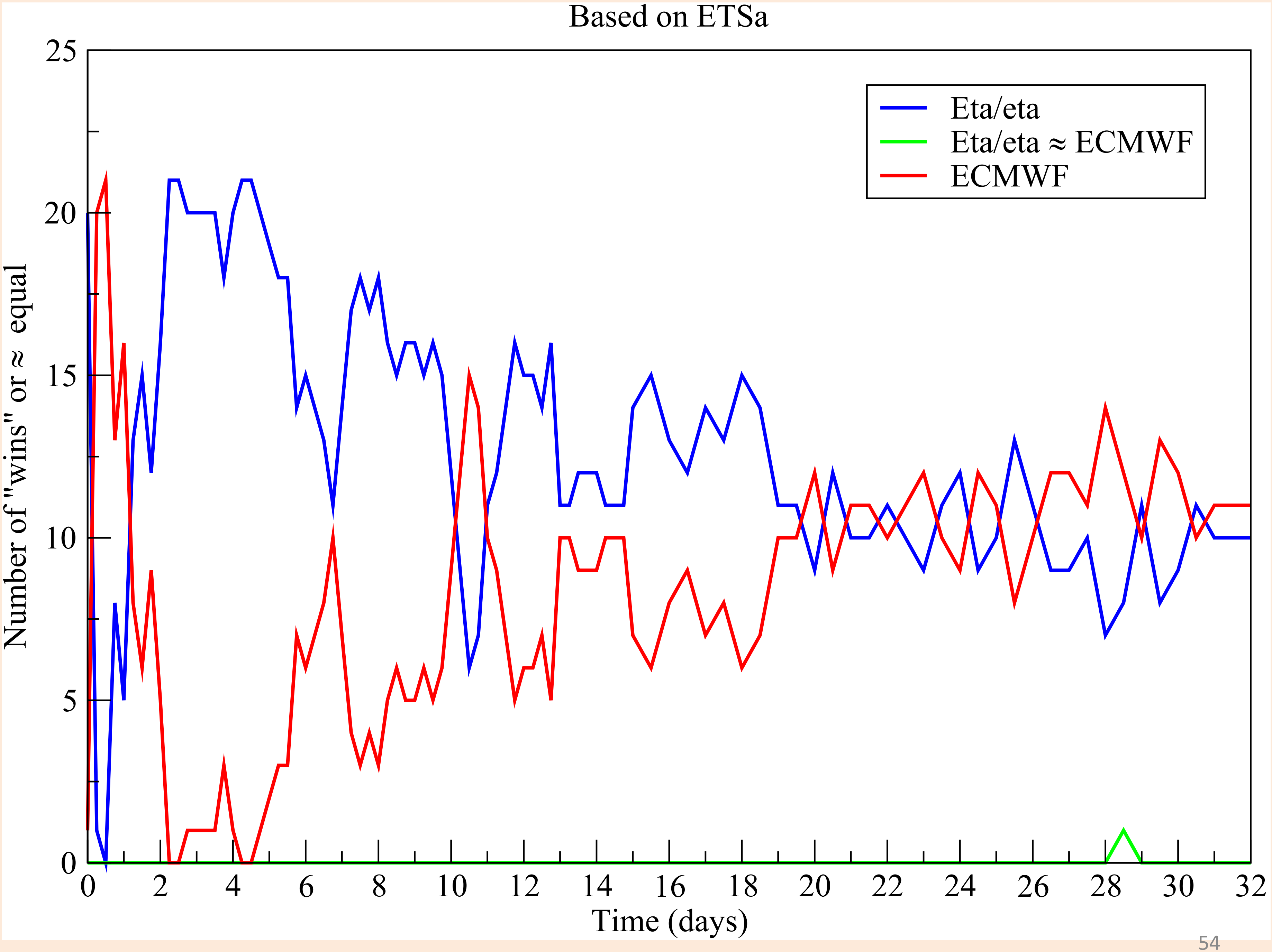
Why was the Eta more accurate at this time ?

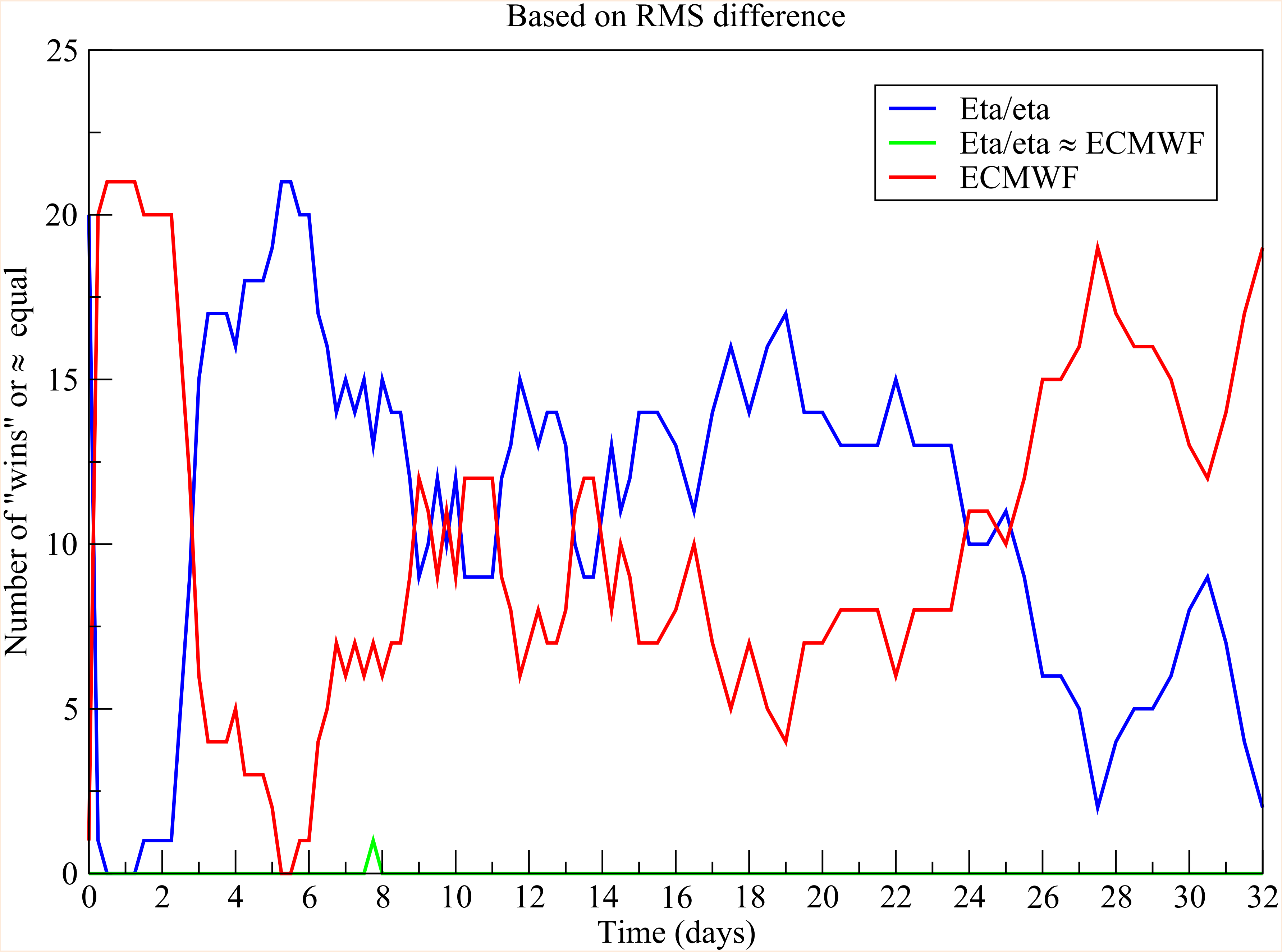


Ensemble average, 21 members, at 4.5 day time: Eta/sigma top left, Eta top right, EC driver bottom left, EC verification analysis bottom right.

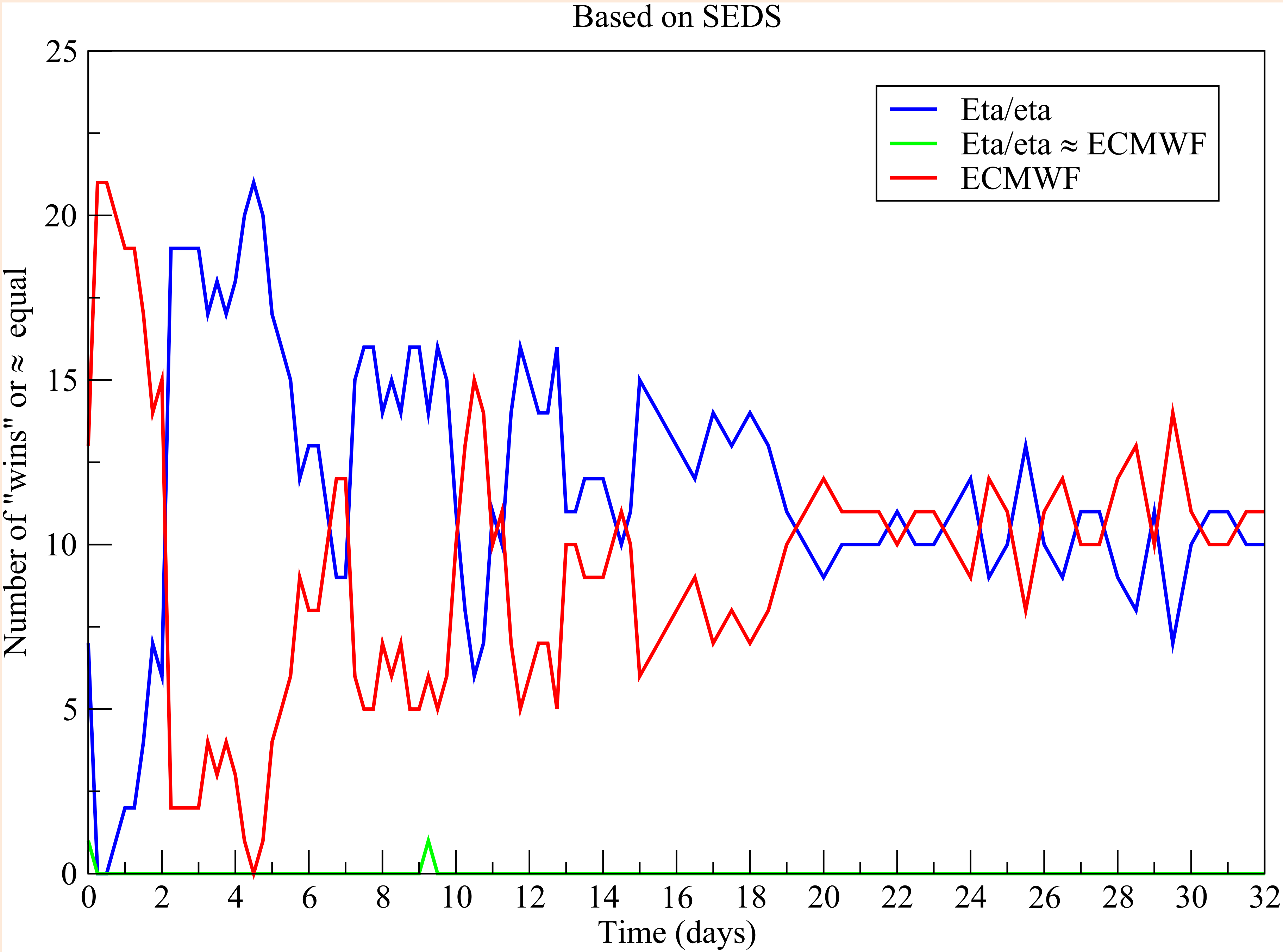
Another way of comparing ensemble model skill

number of “wins”


Based on ETSa
Eta/eta
Eta/eta ≈ ECMWF
ECMWF
Number of "wins" or ≈ equal
Time (days)
0
5
10
15
20
25
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32


Based on RMS difference
Eta/eta
Eta/eta ≈ ECMWF
ECMWF
Number of "wins" or ≈ equal
Time (days)
0
5
10
15
20
25
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32

Other ways of modifying ETS (or, GSS) aimed at reducing the possibility of artificially manipulating the score, in particular by increasing bias; and its non-informative behavior for rare events (Wilks 2011, p. 313); *symmetric extreme dependency score, SEDS*



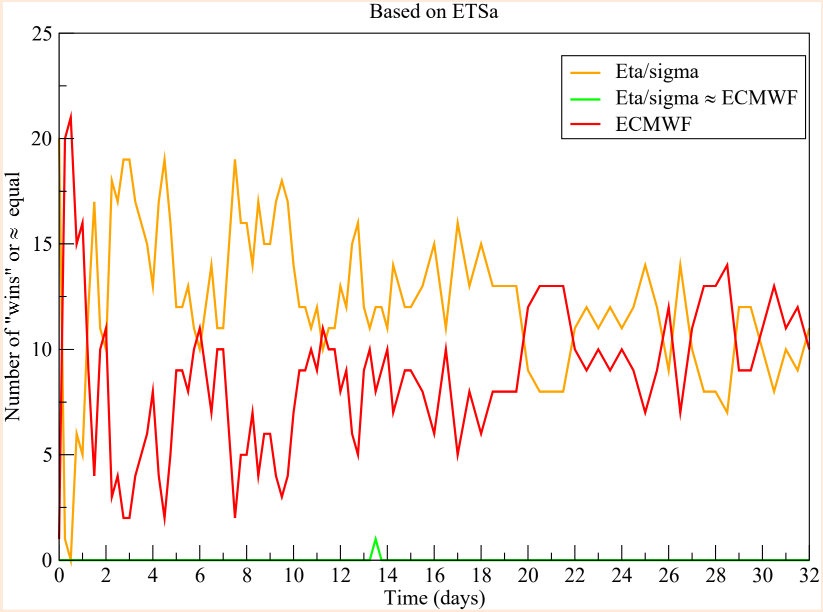
Based on SEDS
Eta/eta
Eta/eta ≈ ECMWF
ECMWF
Number of "wins" or ≈ equal
Time (days)
0
5
10
15
20
25
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32

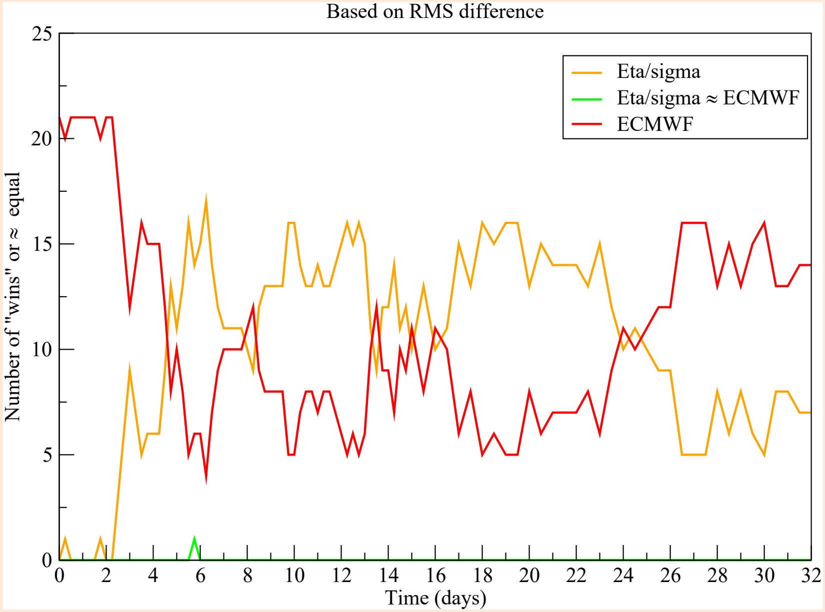
Using each of three accuracy scores, ETSa, RMS difference, and SEDS, at times ranging from 2.25 to 5.5 days, events occurred, 4, 2, 1 times, of all 21 Eta members achieving better scores than their EC driver members

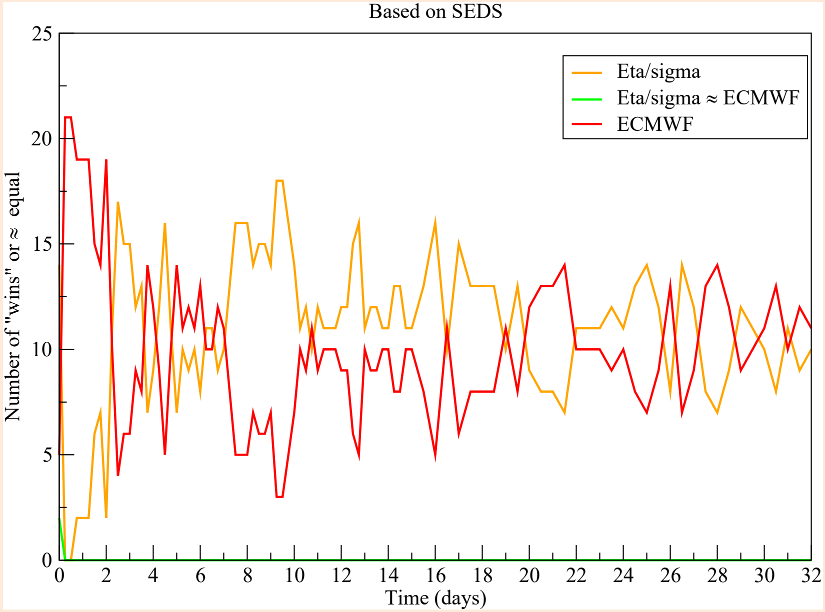
What happens if the Eta is switched to use sigma?

Using each of three accuracy scores, ETSa, RMS difference, and SEDS, at times ranging from 2.25 to 5.5 days, events occurred, 4, 2, 1 times, of all 21 Eta members achieving better scores than their EC driver members

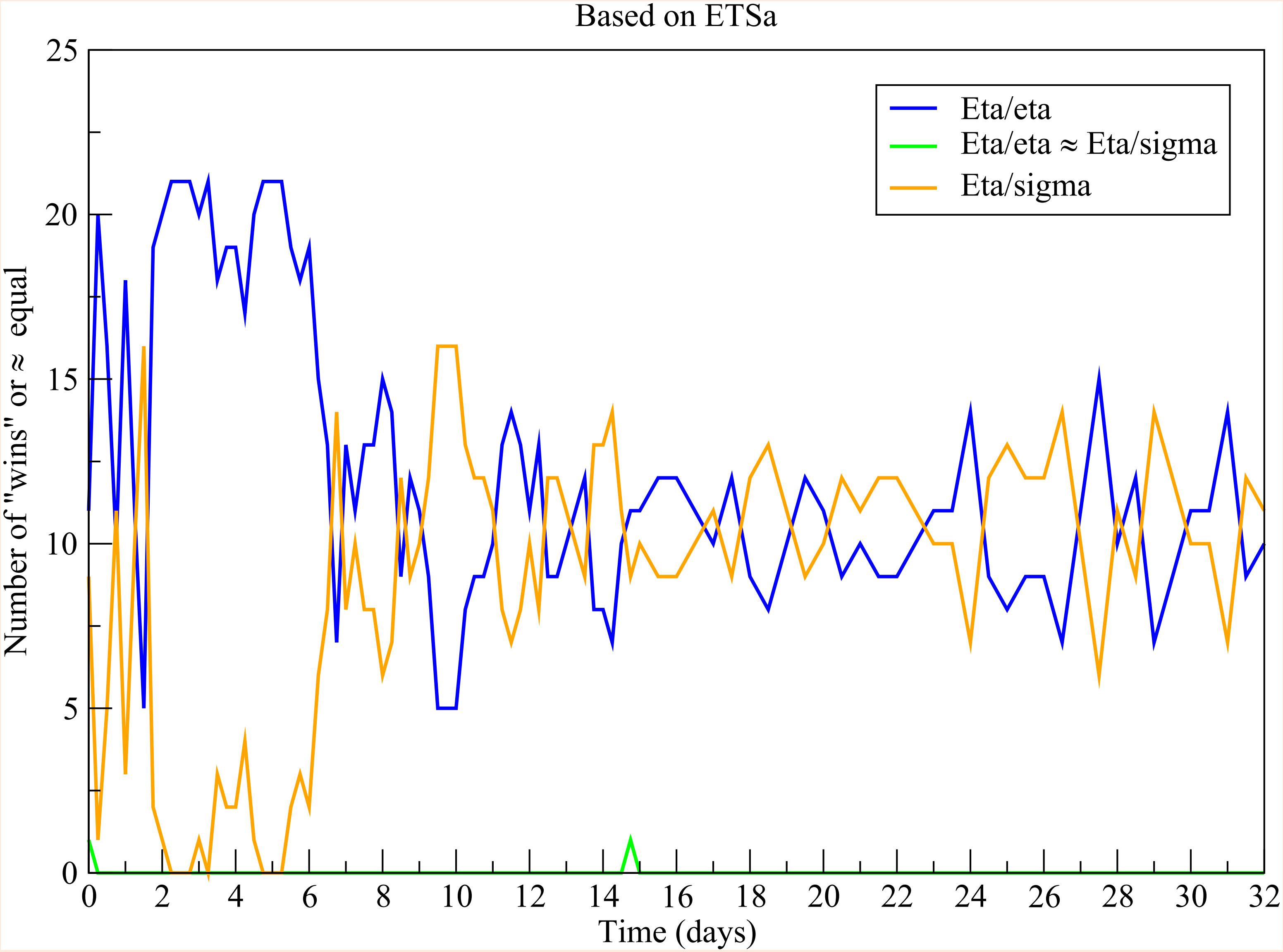
What happens if the Eta is switched to use sigma?

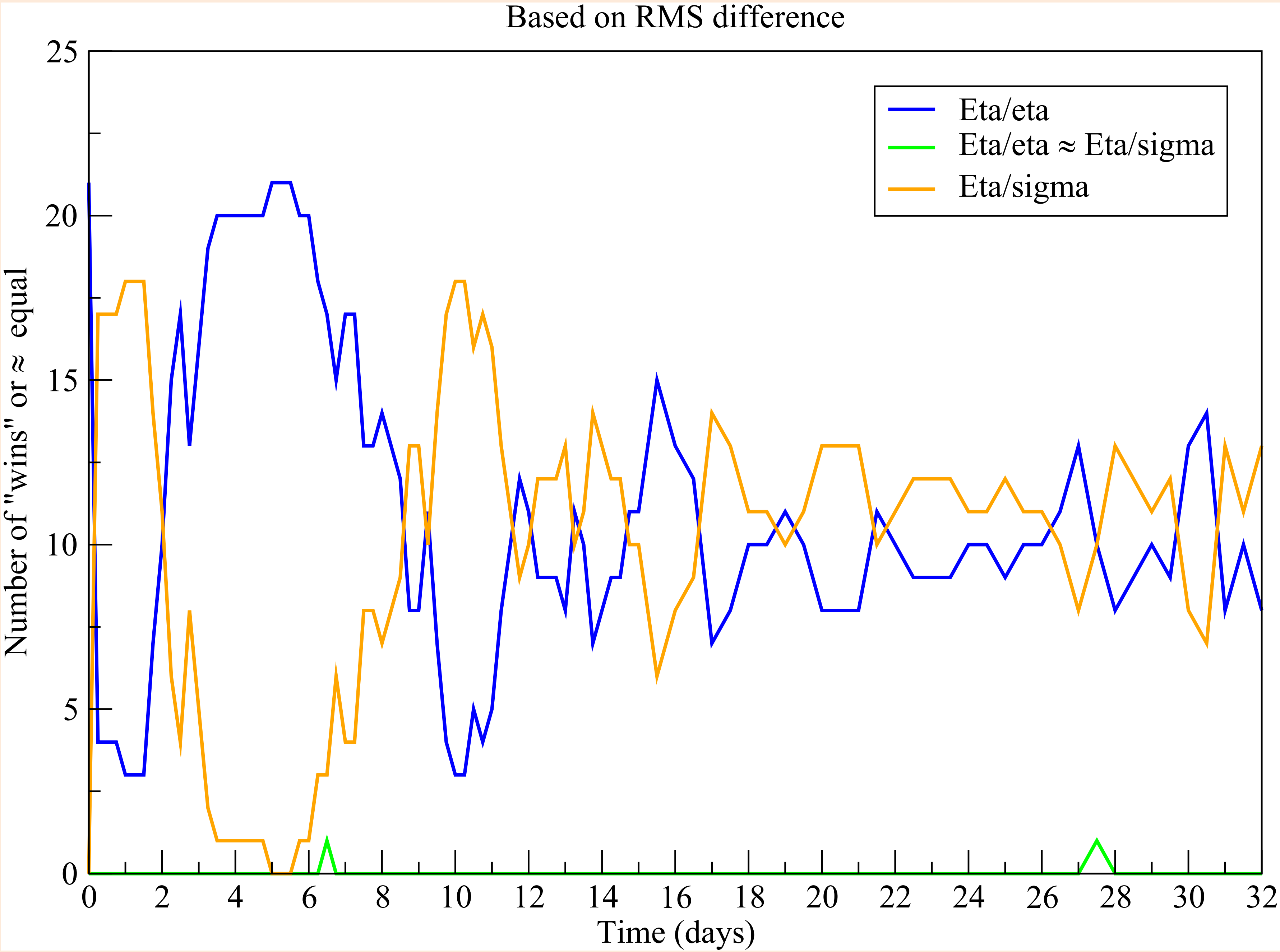


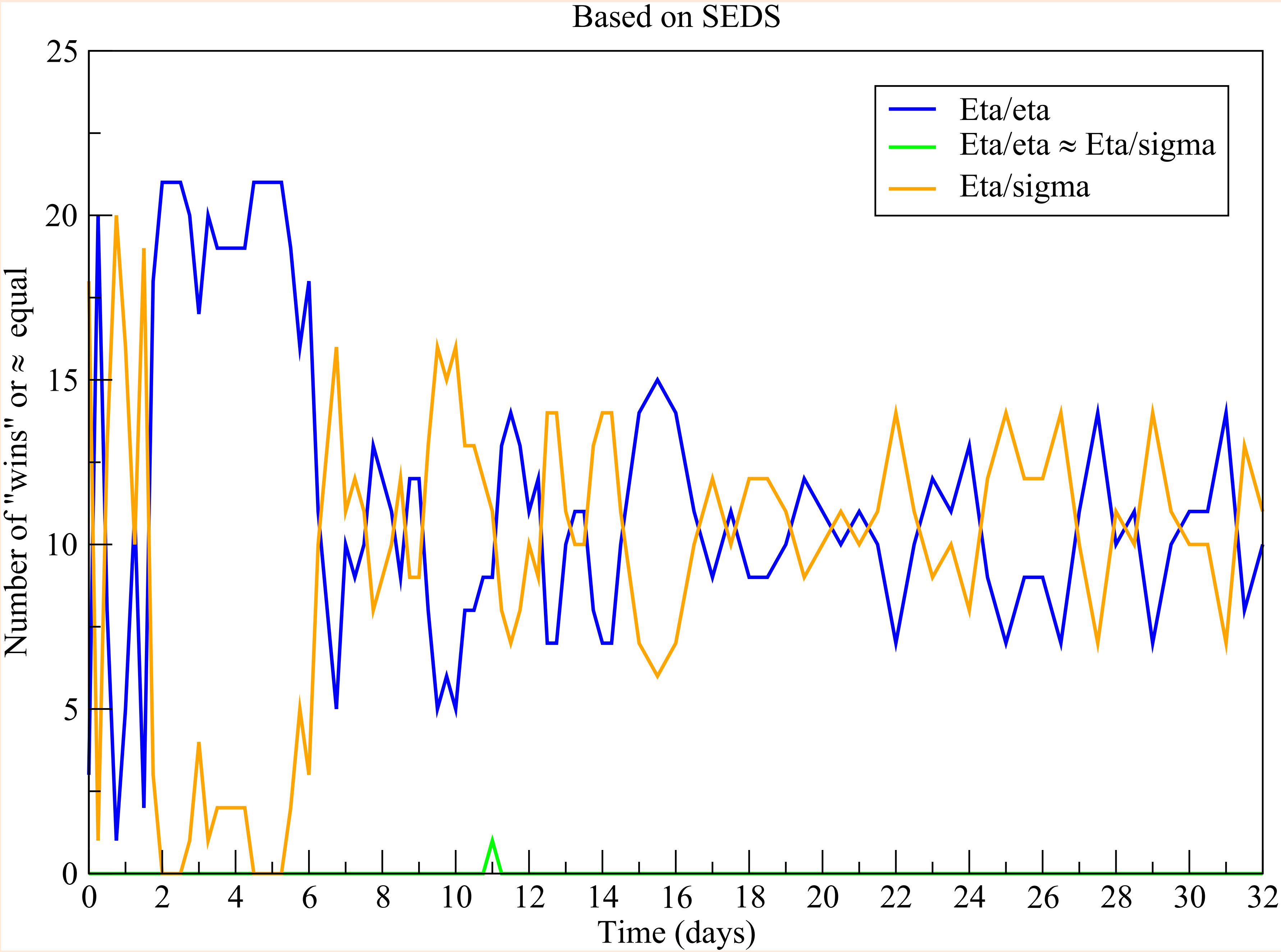




Eta vs. Eta/ sigma:




Based on RMS difference
Eta/eta
Eta/eta ≈ Eta/sigma
Eta/sigma
Number of "wins" or ≈ equal
Time (days)
0
5
10
15
20
25
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32


Based on SEDS
Eta/eta
Eta/eta ≈ Eta/sigma
Eta/sigma
Number of "wins" or ≈ equal
Time (days)
0
5
10
15
20
25
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32

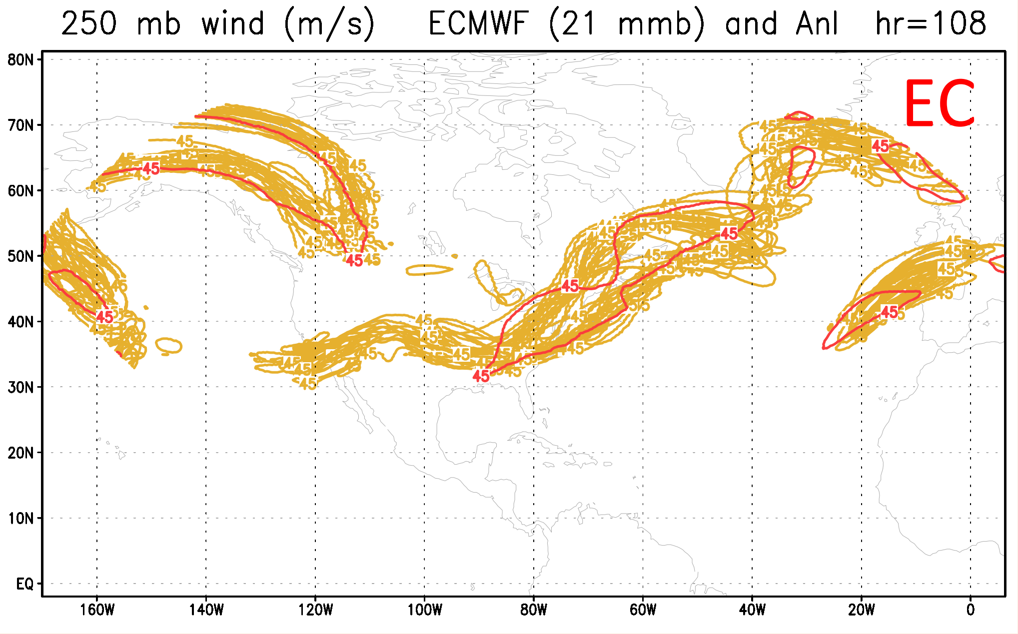
Now:

Contours of all 21 members of areas of wind speeds > 45 m/s

Now:

Contours of all 21 members of areas of wind speeds > 45 m/s

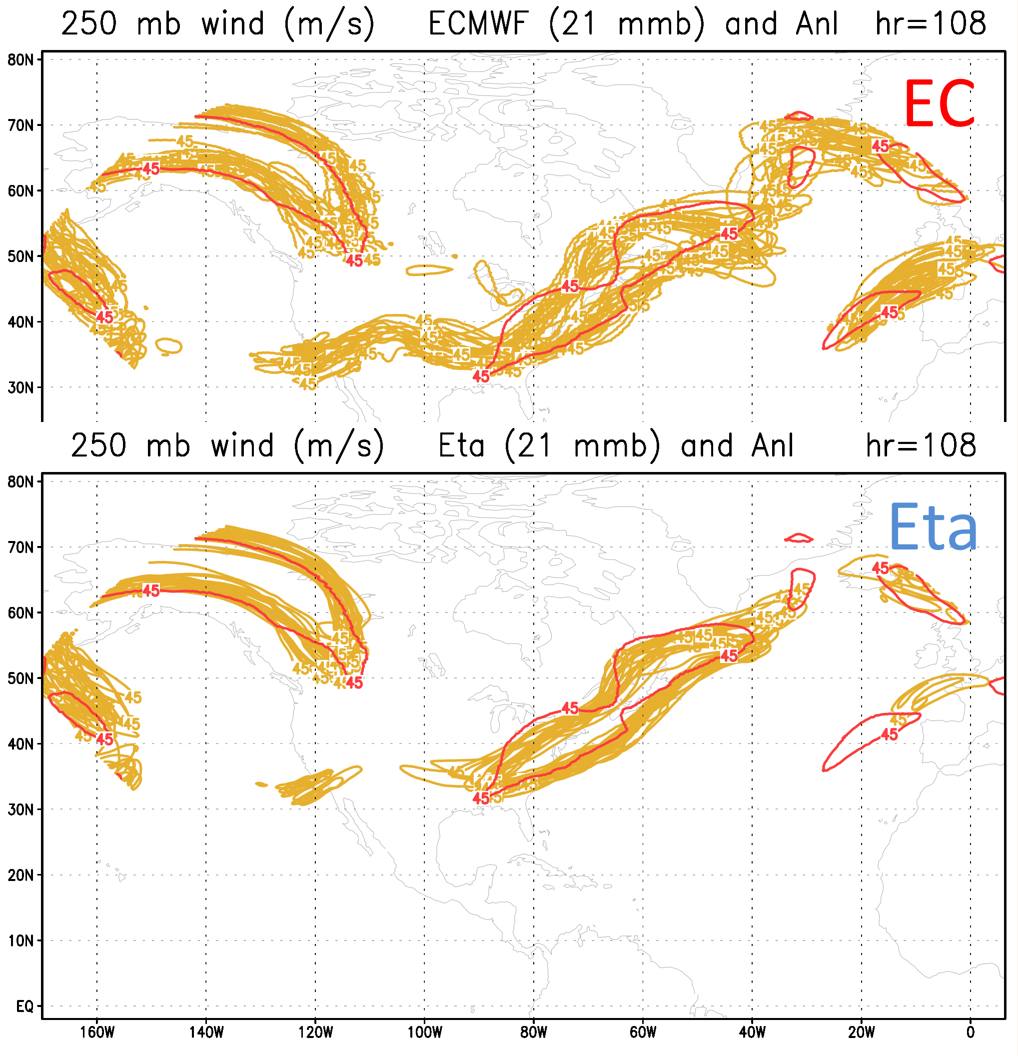
In red are contours of ECMWF verification analysis



Now:

Contours of all 21 members of areas of wind speeds > 45 m/s

In red are contours of ECMWF verification analysis

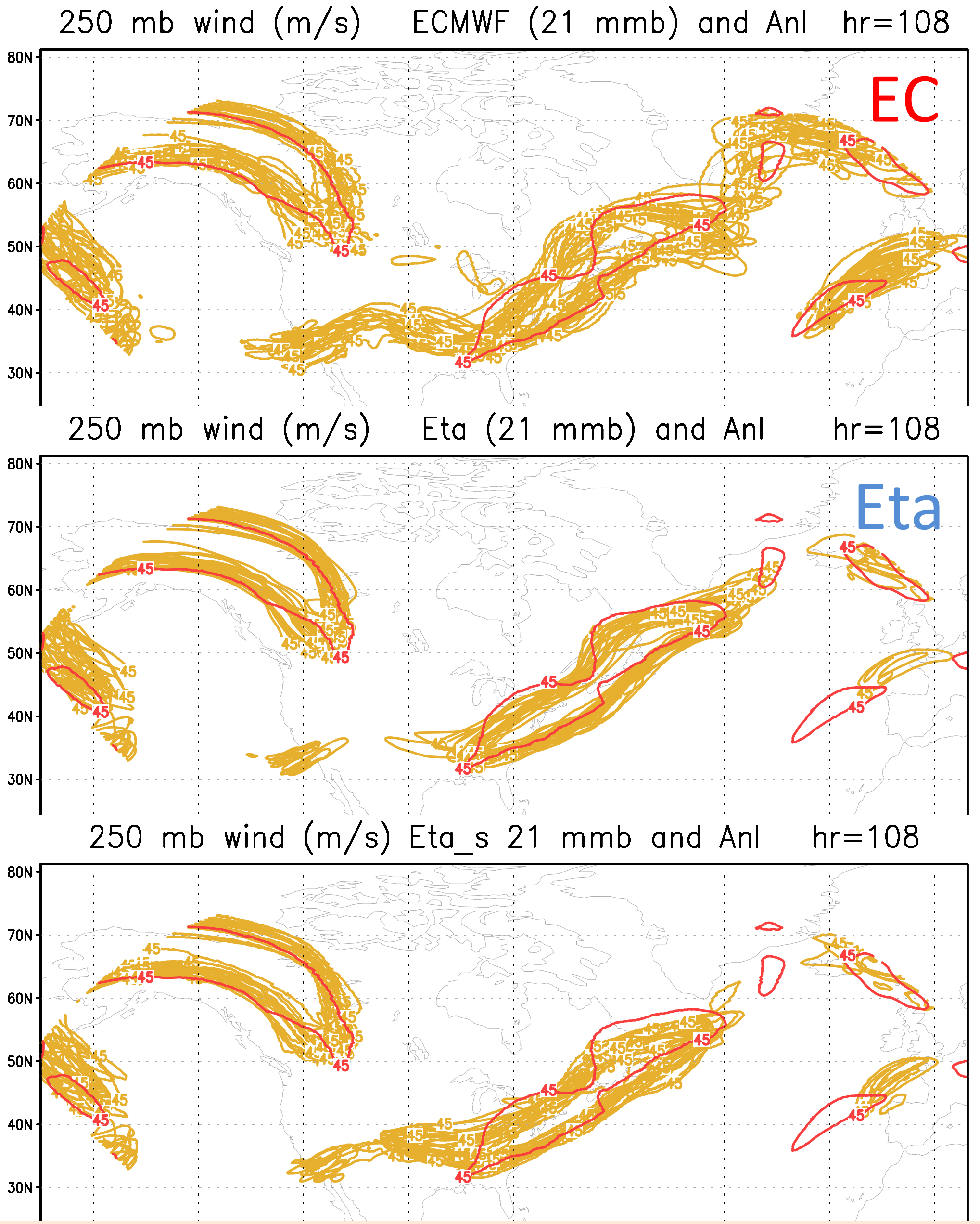


Now:

Contours of all 21 members of areas of wind speeds > 45 m/s

In red are contours of ECMWF verification analysis

Eta/sigma :



Conclusion 1

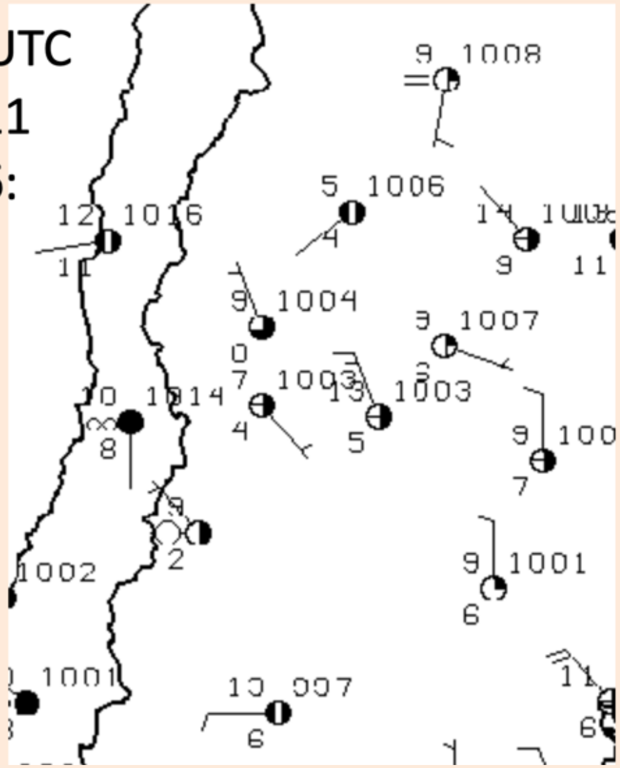
- Strong evidence that coordinate systems intersecting topography performs significantly better than terrain-following systems;

(in agreement with Steppeler et al. 2013)

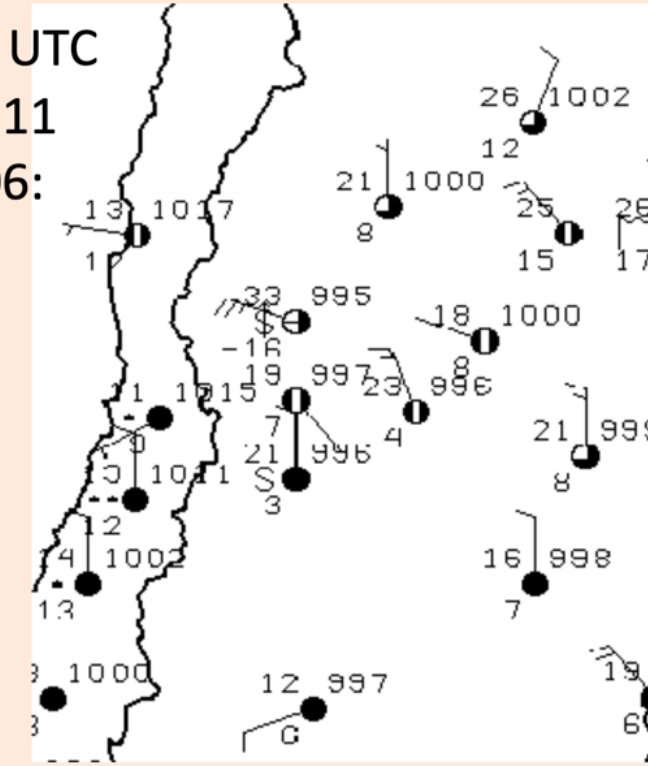
But there must be more reasons / why is the Eta/sigma more accurate than the EC ?

Look at the results of a zonda windstorm case:

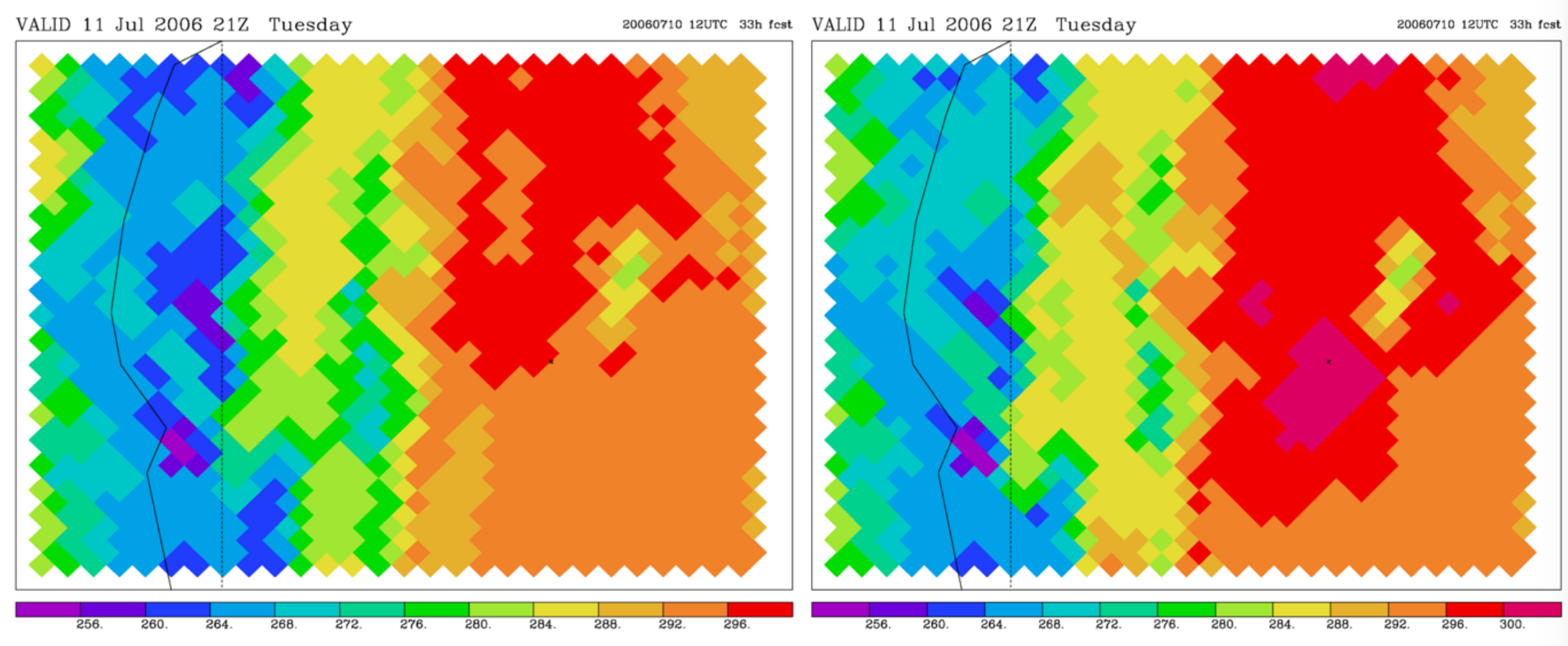
1200 UTC
July 11
2006:



1800 UTC
July 11
2006:



Sections of surface maps illustrating a case of an intense "zonda" windstorm in the lee of the Andes. Warming from 9 to 33°C in 6 h, 24°C, is seen at the station San Juan, 630 m above sea level, close to the middle of the above sections. Valid times are displayed in the top left corner of the maps.



Forecast lowest cell temperatures at 33 h of the case discussed in Section 9 of Mesinger et al. (2012). The left-hand plot shows the result obtained using (3) for both the slantwise and the vertical advection, while the right-hand plot shows the result with these advections replaced by the finite-volume versions. The roughly vertical line on the left sides of the plots is the Chile-Argentina border, while the straight line is the 70°W meridian. The small cross to the right of the centers of plots shows the place of the San Juan station. Warming obtained in 9 h is > 20°C !

Conclusion 2:

Finite-volume vertical advection !

Other candidate reasons:

- Arakawa horizontal advection scheme (Janjić 1984);
- Very careful construction of model topography (MV2017), with grid cell values selected between their mean and silhouette values, depending on surrounding values, and no smoothing;
- Exact conservation of energy in space differencing in transformation between the kinetic and potential energy;
- . . . . . .

Thank you !