

Atmospheric turbulence

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Turbulence? **Exists** (an observed fact !)

Results in very important transports !!

Transports:

- **Advection** / transports by model-resolved motions;
- **Turbulence** transports: transports by turbulence eddies;
- Near the ground surface: **molecular** transports

Presentation:

Treatment of turbulence, PBL/surface layer,
molecular transports/ground surface

1. Turbulence transports/ methods used: *

Horizontal turbulence transports are unimportant/negligible with resolved horizontal scales much larger than the vertical scale - as is the case in regional atmospheric models (e.g., Mellor 1985). Or, Bougeault (1997, p 79): three-dimensional turbulence effects "become important only when the horizontal resolution approaches 1 km". (References at the end)

Thus: *vertical* transports. The typical approach: for a specific variable A (quantity per unit volume: ρA), define "exchange coefficient" K_A by

$$\frac{\partial A}{\partial t} = \frac{\partial}{\partial z} \left(K_A \frac{\partial A}{\partial z} \right) \quad (1)$$

Fundamental task: determine K_A .

* White patches up to slide about 30: Scanned pieces from an ICTP lecture, 1999 (or 2000?): "Boundary layer, turbulence transports, horizontal diffusion", updated. **Original pdf available from FM on request.**

Very large variety of schemes. Some recent references: Bougeault (1997); reports which follow Nielsen (1999).

- Schemes expressing K_A as a function of model-resolved variables (shear, buoyancy / Richardson number, ...) Examples: "Louis" scheme, "Holtslag" scheme, ... (e.g., Louis 1979, Rummukainen 1999, ...)
- Schemes with a prognostic equation for the turbulence kinetic energy (TKE). Popular member: Mellor-Yamada level 2.5
- Schemes with two and more prognostic equations for turbulence quantities. E.g. TKE- ε schemes, Mellor-Yamada level 3 or more, ...

Also: non-local schemes, not following (1), or not only following (1).

Turbulence closure:

- Consider variables consisting of **mean values**, and fluctuations,

e.g.:

$$\tilde{u} = U + u, \quad \tilde{v} = V + v, \quad \dots, \quad (2)$$

- Assume** a set of properties for ensemble averaging ("Reynolds averaging"):

$$\overline{AB} = AB + \overline{ab} \quad \overline{A} = \overline{A} \quad \dots \quad (3)$$

- Assume** that the **mean values** satisfy the governing equations; write them \rightarrow (4)
- Write governing equations for the total values: \rightarrow (5)
- Subtract (4) from (5), to obtain prognostic equations for u, v, \dots (In tensor notation: u_j , and θ) \rightarrow (6).

New variables appear as a result of (3):

$$\overline{u_i u_j} \quad \overline{u_j \theta} \quad (7)$$

"Reynolds stresses". These are the variables we need to describe the effect of turbulence on mean quantities. However, more variables than prognostic equations: the closure problem.

Get prognostic equations for Reynolds stresses by time differentiating (7) and inserting from (6).

However, yet additional new variables:

$$\overline{u_k u_i u_j} \quad \overline{p u_j} \quad \overline{u_i u_j \theta} \quad \overline{p \theta}$$

A variety of assumptions by numerous people.

Mellor and Yamada (1974, 1982):

Assumptions due to Kolmogorov, 1941, and Rotta, 1951. Tensor symmetry properties, dimensional analysis considerations. Analyze terms with respect to order of deviation from isotropy. Introduce systematic simplifications based on the assumption that the degree of anisotropy is small.

--> terms including a variety (five) of length scales

Assumption: all five length scales proportional to a single "master length scale", l

Mellor-Yamada "level 2.5": reduce the problem to just one prognostic equation ("M-Y 2.5"); very popular – many models

$$\frac{d(q^2/2)}{dt} - \frac{\partial}{\partial z} \left(lq S_q \frac{\partial}{\partial z} \left(\frac{q^2}{2} \right) \right) = P_s + P_b - \varepsilon,$$

where

$$q^2 = u^2 + v^2 + w^2,$$

is twice the turbulence kinetic energy. P_s and P_b : shear and buoyancy production, given by

$$P_s = - \overline{wu} \frac{\partial U}{\partial z} - \overline{wv} \frac{\partial V}{\partial z},$$

$$P_b = \beta g \overline{w\theta_v};$$

Dissipation, ε , is given by

$$\varepsilon = \frac{q^3}{B_1 l}.$$

S_q , β , and B_1 above are constants. Exchange coefficients for momentum and heat, K_M and K_H , are

$$K_M = lq S_M, \quad K_H = lq S_H.$$

The "stability functions" S_M and S_H can be calculated from

$$G_M = \frac{l^2}{q^2} \left[\left(\frac{\partial U}{\partial z} \right)^2 + \left(\frac{\partial V}{\partial z} \right)^2 \right],$$

$$G_H = -\frac{l^2}{q^2} \beta g \frac{\partial \Theta_v}{\partial z},$$

via equations that involve additional constants (Mellor and Yamada 1974, 1982; also [Janjic 1990](#)).

The realizability problem: solving for S_M and S_H -- ill-conditioned in a region of the G_M, G_H plane. Janjic (1990, similar to MY 1982):

$$G_H \leq 0.024, \quad G_M \leq 0.36 - 15 G_H. \quad (3.9)$$

More: Helfand and Labraga (JAS, 1988); Galperin et al. (JAS 1988: level 2 $1/4$ scheme).

In the Eta: rather than restrict G_H, G_M , restrict l

Mesinger (1993a), Janjic (1996)

A summary of the problem described in Section 4 of

Mesinger, F., 2010: Several PBL parameterization lessons arrived at running an NWP model. Intern. Conf. Planetary Boundary Layer and Climate Change, IOP Publishing, IOP Conf. Series: Earth and Environmental Science **13** (2010) 012005 doi:10.1088/1755-1315/13/1/012005. (Available online at

<http://iopscience.iop.org/1755-1315/13/1/012005>). Open access :)

How was this discovered? Mellor-Yamada clipping:

$$G_H \leq 0.024, \quad G_M \leq 0.36 - 15G_H$$

Recall:

$$G_M \equiv \left(\frac{l}{q}\right)^2 \left[\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2 \right], \quad G_H \equiv -\left(\frac{l}{q}\right)^2 \beta g \frac{\partial \Theta_v}{\partial z}$$

and:

$$P_s + P_b - \varepsilon = \frac{q^3}{l} \left(S_M G_M + S_H G_H - \frac{1}{B_1} \right)$$

with the Blackadar length scheme (Janjić 1990)

$$l = l_0 k z / (k z + l_0), \quad l_0 = \alpha \int z q dp / \int q dp, \quad \alpha = \text{const}$$

but the more recent code had above the boundary layer

$$l = \min(l_\Delta, l_D, k z), \quad l_\Delta = c_s \Delta z \quad l_D = c_r q / N$$

A person at NCAR, A. Marroquin complained to me of no turbulence above the PBL. He used the Blackadar definition of l only, thus, large l !!

The model wanted to generate turbulence, but the G_M, G_H were apparently clipped and on top of it the TKE total production was being divided by a large Blackadar l :

$$P_s + P_b - \varepsilon = \frac{q^3}{l} \left(S_M G_M + S_H G_H - \frac{1}{B_1} \right)$$

No turbulence above the boundary layer :(

However, switching to the PBL scheme of the bottom line of the previous slide: credibly looking upper troposphere turbulence! :)

Serendipity ?

Thus, Eta: Remove clipping of the G_M, G_H / instead,
enforce $G_H < 0.24$ by reducing ℓ !

4. The surface layer: Monin-Obukhov similarity

"Surface layer": shallow layer where *the turbulent fluxes differ little from their surface value*. Extending from the ground to some meters above. ("Some meters": 5 to 50 m)

Also "constant flux layer". Warning: this is precisely the layer in which the turbulent fluxes change most rapidly!

"Atmospheric surface layer", ASL

Basic notation. Consider the "neutral" case first: heat transport not having a significant impact. (Always near the surface).

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Relevant variables: height, z , and "friction velocity", u_* , defined by

$$u_* u_* = - \overline{u'w'}$$

Also: "velocity scale".

Momentum profile:

$$\frac{du}{u_*} = \frac{dz}{l}$$

where

$$l = kz$$

is a characteristic length scale, or eddy size. k : von Karman constant, 0.4. For traditional reasons, we have here changed notation to use lower case u for the mean velocity. Integration leads to

$$u = \frac{u_*}{k} \ln \frac{z}{z_0}. \quad (8)$$

"The logarithmic wind profile". z_0 : "roughness length". However: "roughness length for momentum", z_{0u} , better.

Stratification (Monin-Obukhov):

Sensible heat flux, $-\overline{w'\theta'}$, is relevant. Traditional: define "temperature scale", θ_* , by

$$\theta_* u_* = -\overline{\theta'w'}$$

Using this temperature scale (in fact, sensible heat flux), Obukhov length is defined, e.g., by

$$L = \frac{u_*^2 \Theta}{kg\theta_*} \quad (9)$$

Nondimensional height can now be formed, z/L , and instead of the

$\frac{du}{dz} = \frac{u_*}{kz}$, etc., MO similarity states that

$$\frac{du}{dz} = \frac{u_*}{kz} \Phi_u\left(\frac{z}{L}\right), \quad \frac{d\Theta}{dz} = \frac{\theta_*}{kz} \Phi_\theta\left(\frac{z}{L}\right), \quad (10)$$

etc. $\Phi_u, \Phi_\theta, \dots$: functions obtained from measurements. "Empirical functions".

To compute fluxes, we need the exchange coefficients K_M and K_H , defined by

$$-\overline{u'w'} = K_M \frac{du}{dz}, \quad -\overline{\theta'w'} = K_H \frac{d\Theta}{dz} \quad (11)$$

Solving (10) not straightforward. Highly implicit. Note: L is a function of the momentum and sensible heat flux, given by K_M and K_H , which we want to obtain. However: standard methods. Two elevations needed;

usually, iterations to solve (10), started with first guess fluxes. First guess fluxes obtained from first guess "bulk" exchange coefficients (coefficients of the finite difference forms of (11)). From the first guess fluxes, obtain first guess L . Several iterations.

What about very close to the ground?

Molecular transports take over !!

In the Eta:

Different over **land** (and **ice**) and over **water**

Over land (and ice):
Account for roughness
elements:

Zilitinkevich (1995):

$$z_{0T} = z_{0m} e^{-A_0 \sqrt{u_* z_{0m} / \nu}}$$

Over water:
Molecular sublayer

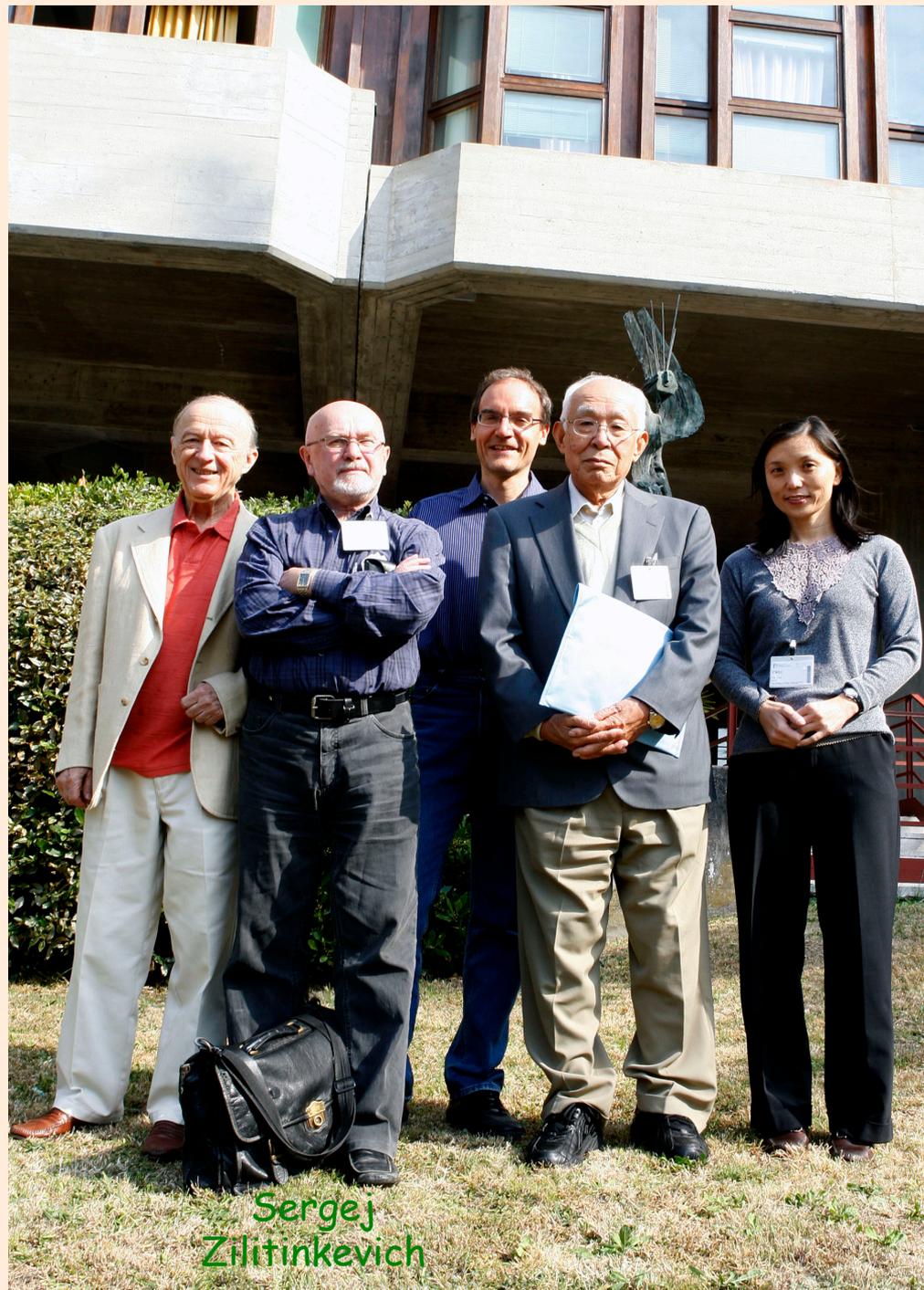
Liu, Katsaros, Businger
(1979, "LKB");
Janjić (1994);

also: Mesinger et al. (2012)
("An upgraded version of
the Eta model")

ICTP, Trieste

2008

Picture credit:
ICTP Photo Archives,
Massimo Silvano



Sergej
Zilitinkevich

Molecular sublayer:

according to measurements of Mangarella et al. (1973):

Three regimes: smooth, rough, and rough with spray;

The flow switches from one to the other according to the value of “roughness Reynolds number”, Rr

$$Rr = u_* z_0 / \nu$$

Seems to work well, an example:

Typhoon Yancy (Aug. 1990)

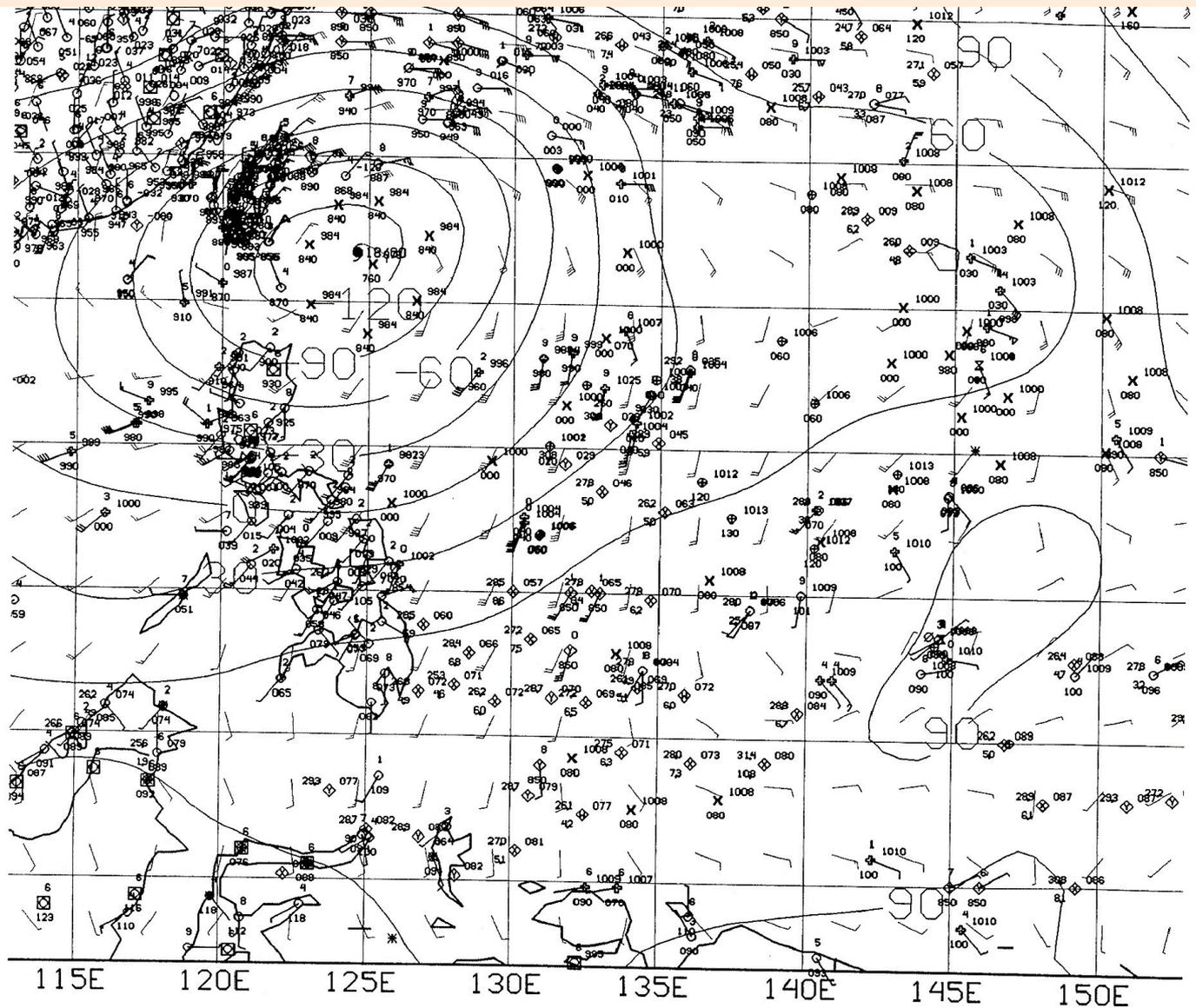
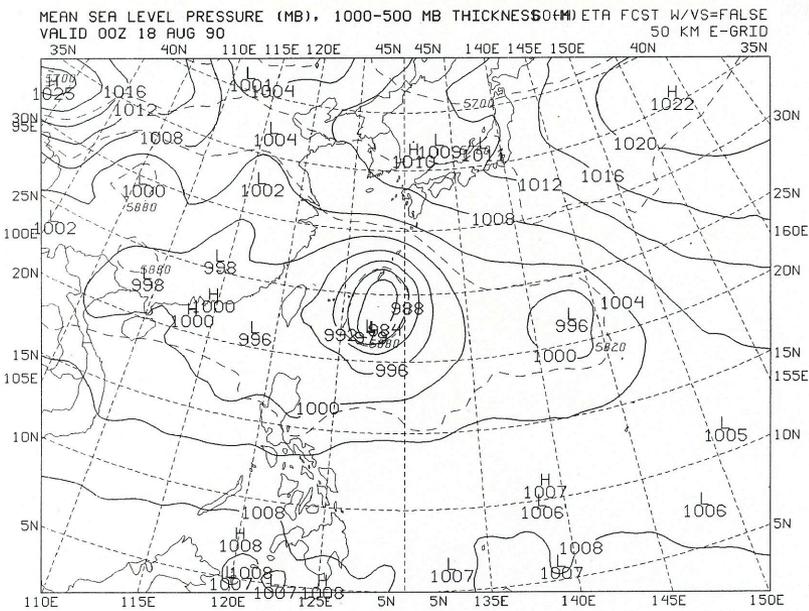


Figure 3. A section of the NMC TCM-90 surface analysis valid 0000 UTC 18 August 1990 (Courtesy of Eric Rogers). Contours of analyzed 1000 mb geopotential heights, in meters, and winds, in knots, are shown; as well as various observations.

Without
molecular
sublayer:



With
molecular
sublayer:

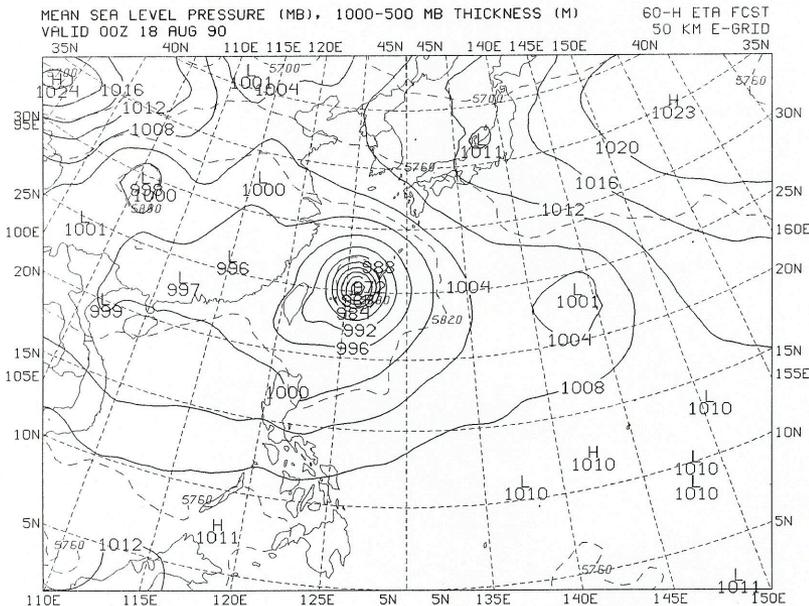


Figure 4. The Eta Model 60-h simulation of the sea level pressure, millibars, and 1000-500 mb thickness, meters, valid 0000 UTC 18 August 1990, with no parameterization of the molecular sublayer, upper panel; same except for the parameterization of the molecular sublayer being included, lower panel. (Courtesy of Eric Rogers.)

More recent effort:

Have molecular sublayer thickness depend on roughness
Reynolds number, based on experimental data compiled
by Brutsaert

At the top of the molecular sublayer, molecular transports must be equal to the turbulent transports:

$$\begin{aligned} \nu \frac{U_1 - U_s}{z_{1u}} &= u_* u_*, \\ \kappa \frac{\Theta_1 - \Theta_s}{z_{1\theta}} &= \theta_* u_*, \\ \varepsilon \frac{q_1 - q_s}{z_{1q}} &= q_* u_*, \end{aligned} \tag{8.1}$$

where ν , κ , and ε are the kinematic viscosity, thermal diffusivity, and molecular diffusivity of water vapor, respectively; u_* is the friction velocity, and θ_* and q_* are analogously defined scaling parameters for the sensible heat and moisture fluxes, respectively. The right hand sides of (8.1) can also be expressed in terms of the standard surface layer bulk relationships, and the equations thus obtained solved for U_1 , Θ_1 and q_1 provided sublayer thicknesses are known. These were obtained by Janjic by postulating

$$\frac{z_{1u} u_*}{C\nu} = \frac{z_{1\theta} u_*}{S\kappa} = \frac{z_{1q} u_*}{D\varepsilon} = \zeta, \tag{8.2}$$

It was considered by Janjic adequate to keep ζ a constant. For $Rr \approx 1$ ^{*} one obtains

$$\zeta = 0.35 \quad (29)$$

Used in the “standard” (or, NCEP) Eta

* Recall: Roughness Reynolds number : $Rr = u_* z_0 / \nu$

As opposed to having ζ constant, a relationship resulting from experimental data (Brutsaert 1982, Fig. 4.1) can be used:

A question can be

asked: if the linear profile at the bottom of the viscous sublayer is linearly extrapolated upwards, and the logarithmic profile of the surface layer is at the same time logarithmically extrapolated downwards, at what elevation will the two extrapolated profiles intersect? This should be the appropriate value of z_{1u} , from which ζ can be calculated.

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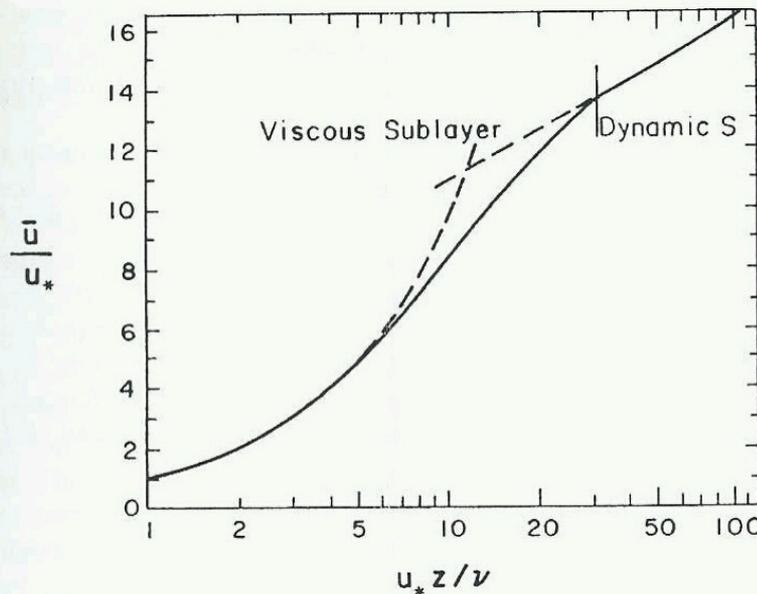


Fig. 4.1. The mean velocity profile in turbulent flow over a smooth surface as observed experimentally. The solid curve describes (4.123) for $z_+ < 5$, and (4.3) with (4.9) for $z_+ > 30$.

One obtains

$$\zeta = 11 / (M Rr^{1/4})$$

The model knows what is Rr :

Relation originally due to Charnock widely used; in the Eta:

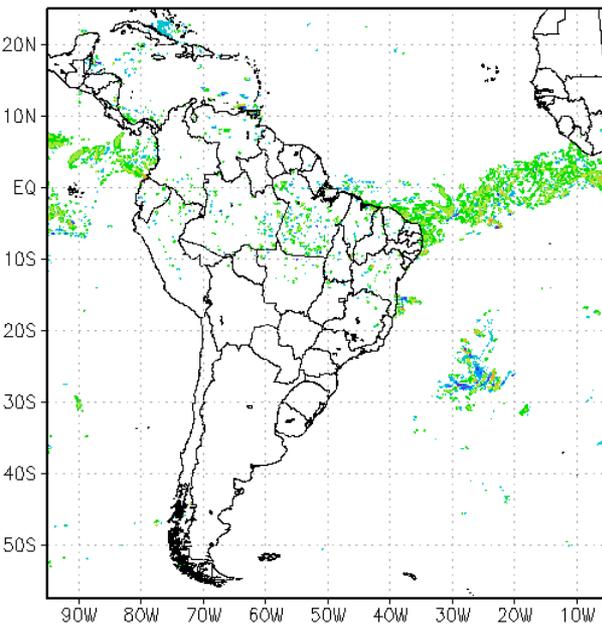
$$z_0 = \frac{0.11\nu}{u_*} + \frac{0.018u_*^2}{g}$$

0.018: *the Charnock constant*; for further reading see e.g., Garratt (1992, pp. 98-100).

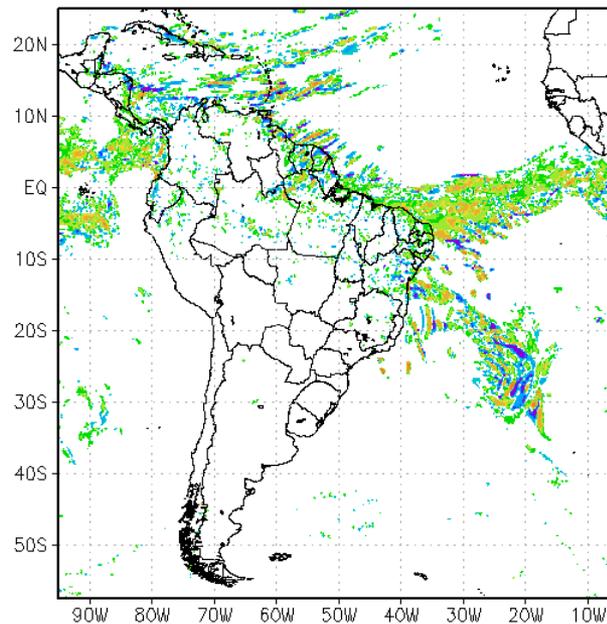
Thus, ζ can be calculated as a function of $Rr = u_* z_0 / \nu$
using the Brutsaert relation (previous slide)

Experiments done by [Josiane Bustamante](#), INPE:

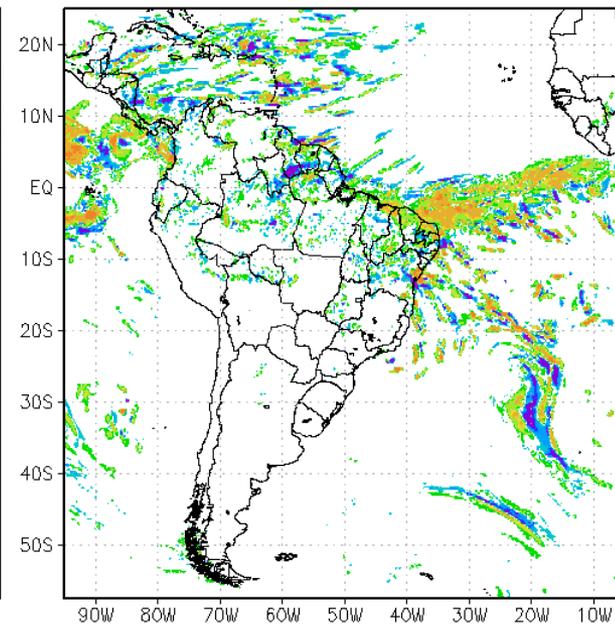
cntrl-cvsc T+24h



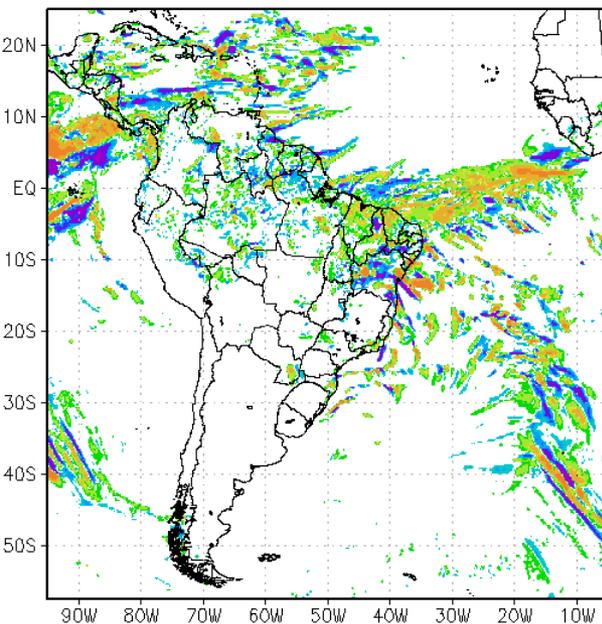
cntrl-cvsc T+48h



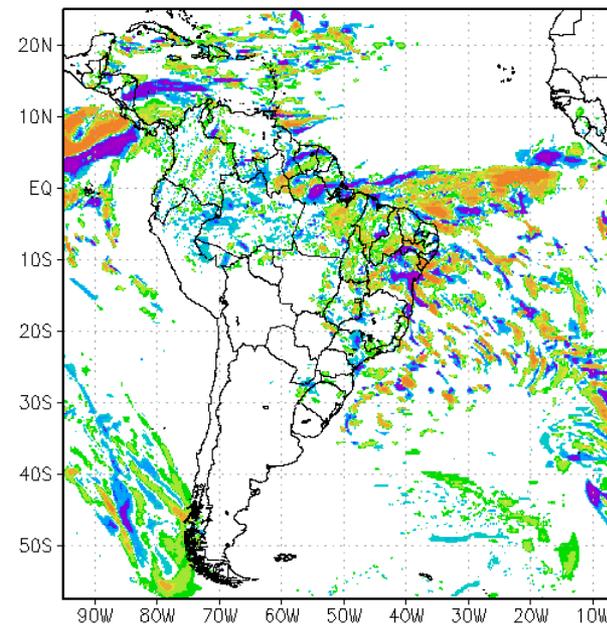
cntrl-cvsc T+72h



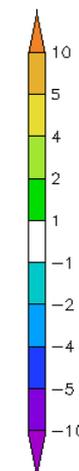
cntrl-cvsc T+96h



cntrl-cvsc T+120h



Difference,
accumulated
precip,
mm / 24 h,
initial condition
27 Apr 2009



Other length scale refinements of the Eta code compared to MY82 and Janjić (1990):

Mesinger, F., 1993b: Sensitivity of the definition of a cold front to the parameterization of turbulent fluxes in the NMC's Eta Model. Research Activities in Atmospheric and Oceanic Modelling, WMO, Geneva, CAS/JSC WGNE Rep. 18, 4.36-4.38:

turbulence closure scheme. As the first change, instead of calculating the model's "Blackadar scheme" values of l , l_b for the model interface elevations (e.g., Janjić 1990), l_b was calculated at layer mid-points and then two-point averaged to obtain the interface values

$$l_{b,k} = (l_{b,k+1/2} + l_{b,k-1/2}) / 2 . \quad (3.1)$$

The second change consisted of the introduction of the frequently used stability restriction on l according to the dimensional formula of Deardorff (1976)

$$l_s = c_T q / N. \quad (2)$$

Here q is the turbulence speed and N is the Brunt-Väisälä frequency. In addition, the use of the Blackadar scheme for l was restricted to the boundary layer, assuming the boundary layer depth to be proportional to the Blackadar's asymptotic length scale l_∞

$$z_i = c_g l_\infty. \quad (3)$$

Note that (3) was previously used by Gambo (1978) and Yamada (1979), but to obtain l_∞ for a given z_i . For interfaces below z_i , we have now defined l as

$$l = \min(l_b, l_s). \quad (4)$$

Above the boundary layer, we have defined l as

$$l = \min(l_s, l_d, kz), \quad (5)$$

where

$$l_d = c_s \Delta z, \quad (6)$$

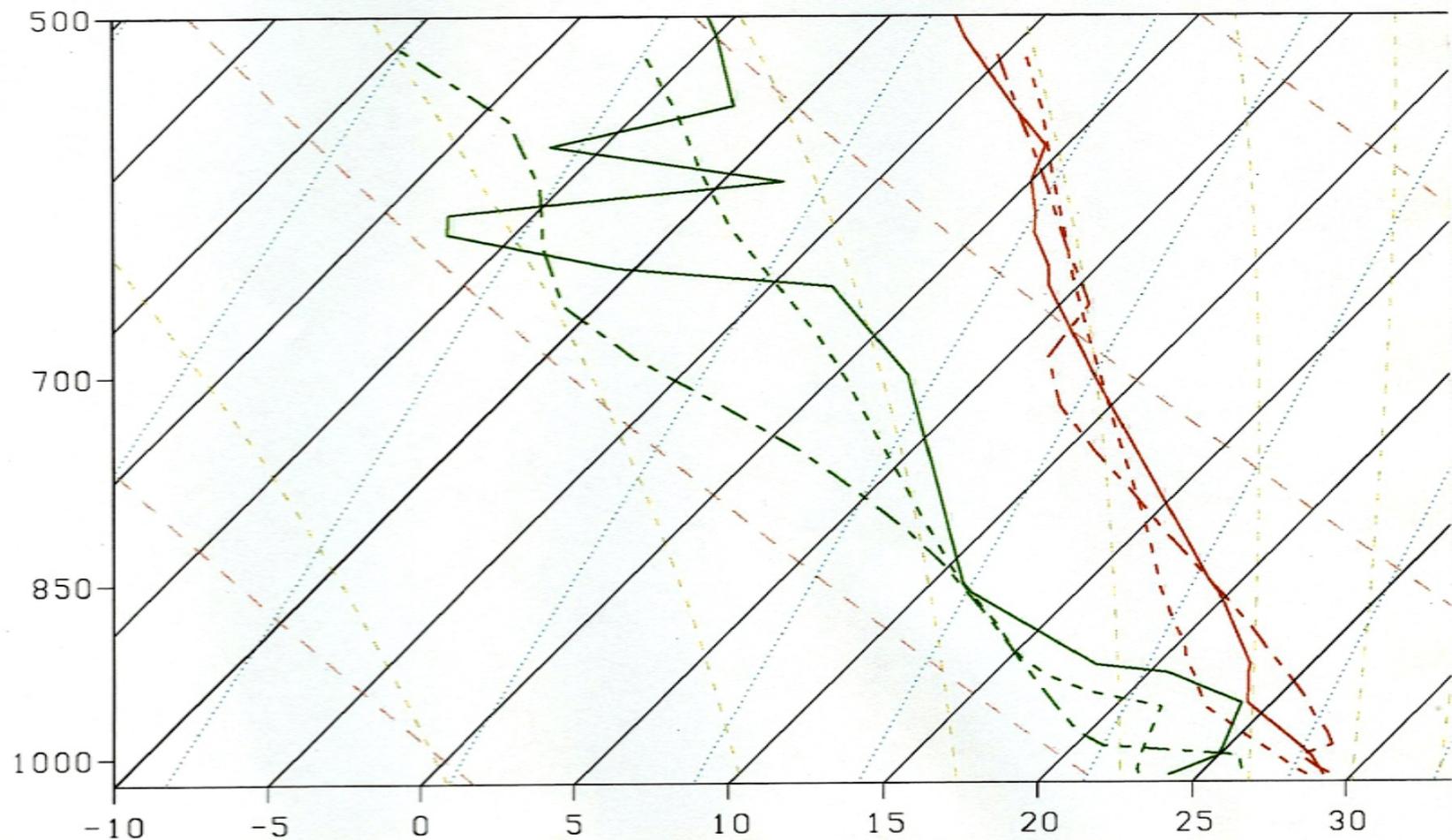


Figure 5. Observed dew point and temperature soundings (solid lines), and same except 24-h forecasts by two versions of the Eta model, one which did not use the length scale averaging scheme (3.1), long/short dashes; and another which did, short dashes; all verifying 0000 UTC 31 July 1997. Temperature, in degrees C, is on the abscissa, and pressure, in mb, on the ordinate; dew point lines are to the left of the corresponding temperature lines. (Joe Gerrity, personal communication.)

- *Horizontal diffusion: do we need it / what is it?*

Jablonowski, C. and D. L. Williamson (2011): The Pros and Cons of Diffusion, Filters and Fixers in Atmospheric General Circulation Models. Chapter 13 in: Lauritzen, P. H., C. Jablonowski, M. A. Taylor, R. D. Nair (Eds.), Numerical Techniques for Global Atmospheric Models, Lecture Notes in Computational Science and Engineering, Springer, Vol. 80, 381-493:

As pointed out by Mellor (1985) the horizontal diffusivities in use by GCMs are typically many orders of magnitude larger than those which would be appropriate for turbulence closures. Thus, horizontal diffusion used by most models cannot be considered a representation of turbulence but should be viewed as a substitute mechanism for unresolved horizontal advective processes.

- What should we (the Eta community) do next?

An “improved Mellor-Yamada turbulence closure model” or “Mellor-Yamada-Nakanishi-Niino” is gaining in popularity. Series of papers, 2001-2009.

Also:

Kitamura, Y., 2010: Modifications to the Mellor-Yamada-Nakanishi-Niino (MYNN) model for the stable stratification case. *J. Meteor. Soc. Japan*, 88, 857-864. doi:10.2151/jmsj.2010-506.

“M-MYNN”

What does the MYNN do ?

- Generate a data base running LES experiments (**resolution 4 m in all three directions !!!**);
- Expand the MY2.5 by not neglecting several terms that MY neglected;
- Update values of five MY empirical coefficients using their LES data base, and evaluate three new coefficients that MY2.5 does not have since their terms were neglected;
- Invent prescription of the length scale dependence on distance from the ground, PBL turbulence, buoyancy;
- Check the performance of the scheme in well-known experimental data (Businger et al., 1971; Wangara Day 33);

From Janjić (MWR 1990): The Level 2.5 turbulence closure model is governed by the equations (MY82):

$$d(q^2/2)/dt - (\partial/\partial z)[lqS_q(\partial/\partial z)(q^2/2)]$$

$$P_s = -\overline{wu}(\partial U/\partial z) - \overline{wv}(\partial V/\partial z), \quad = P_s + P_b - \epsilon, \quad (3.1)$$

$$P_b = \beta g \overline{w\theta_v}, \quad \epsilon = q^3(B_1 l)^{-1}, \quad (3.2)$$

$$-\overline{wu} = K_M \partial U/\partial z, \quad -\overline{wv} = K_M \partial V/\partial z, \quad (3.3_1)$$

$$-\overline{w\theta_v} = K_H \partial \Theta_v/\partial z, \quad -\overline{wS} = K_H \partial S/\partial z, \quad (3.3_2)$$

$$K_M = lqS_M, \quad K_H = lqS_H, \quad (3.4)$$

$$S_M(6A_1A_2G_M) + S_H(1 - 3A_2B_2G_H - 12A_1A_2G_H) = A_2, \quad (3.5_1)$$

$$S_M(1 + 6A_1^2G_M - 9A_1A_2G_H) - S_H(12A_1^2G_H + 9A_1A_2G_H) = A_1(1 - 3C_1), \quad (3.5_2)$$

$$G_M = l^2q^{-2}[(\partial U/\partial z)^2 + (\partial V/\partial z)^2],$$

$$G_H = -l^2q^{-2}\beta g \partial \Theta_v/\partial z. \quad (3.6)$$

$$S_M(6A_1A_2G_M)$$

$$+ S_H(1 - 3A_2B_2G_H - 12A_1A_2G_H) = A_2, \quad (3.5_1)$$

$$S_M(1 + 6A_1^2G_M - 9A_1A_2G_H)$$

$$- S_H(12A_1^2G_H + 9A_1A_2G_H) = A_1(1 - 3C_1), \quad (3.5_2)$$

$$G_M = l^2q^{-2}[(\partial U/\partial z)^2 + (\partial V/\partial z)^2],$$

$$G_H = -l^2q^{-2}\beta g\partial\Theta_v/\partial z. \quad (3.6)$$

A_1, A_2, B_1, B_2, C_1 : empirical constants

Janjić (NCEP Office Note 437): constants different from MY82

Nakanishi, Niino: **New values of constants**, including values for **constants within terms not included in MY82**

Based on several measurements, Mellor and Yamada (1982) estimated the closure constants as

$$\begin{aligned}(A_1, A_2, B_1, B_2, C_1) &= (0.92, 0.74, 16.6, 10.1, 0.08), \\ (C_2, C_3, C_4, C_5) &= (0, 0, 0, 0).\end{aligned}\tag{9}$$

NN09:

we have a new set of the closure constants as

$$\begin{aligned}(A_1, A_2, B_1, B_2, C_1) & \\ &= (1.18, 0.665, 24.0, 15.0, 0.137), \\ (C_2, C_3, C_4, C_5) & \\ &= (0.75, 0.352, 0.0, 0.2).\end{aligned}\tag{66}$$

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equation for L that consists of three length scales, L_S , L_T , and L_B , i.e.,

$$\frac{1}{L} = \frac{1}{L_S} + \frac{1}{L_T} + \frac{1}{L_B}, \quad (38)$$

where L_S is the length scale in the surface layer as obtained in Section 4.1, L_T the length scale depending upon the turbulent structure of the PBL (Mellor and Yamada, 1974), and L_B the length scale limited by the buoyancy effect. This expression is designed in order that the shortest length (or time) scale may control L . L_S , L_T , and L_B are given by

$$L_S = \begin{cases} kz/3.7, & \zeta \geq 1 \\ kz(1 + 2.7\zeta)^{-1}, & 0 \leq \zeta < 1 \\ kz(1 - \alpha_4\zeta)^{0.2}, & \zeta < 0, \end{cases} \quad (39)$$

$$L_T = \alpha_1 \frac{\int_0^\infty qz dz}{\int_0^\infty q dz}, \quad (40)$$

$$L_B = \begin{cases} \alpha_2 q/N, & \partial\Theta/\partial z > 0 \text{ and } \zeta \geq 0 \\ [\alpha_2 q + \alpha_3 q(q_c/L_T N)^{1/2}]/N, & \partial\Theta/\partial z > 0 \text{ and } \zeta < 0 \\ \infty, & \partial\Theta/\partial z \leq 0, \end{cases} \quad (41)$$

where α_1 , α_2 , α_3 , and α_4 are empirical constants, and $q_c \equiv [(g/\Theta_0)\langle w\theta \rangle_g L_T]^{1/3}$ is a velocity scale similar to the convective velocity w_* .

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- Expand the MY2.5 by not neglecting several terms that MY neglected;
- Update values of five MY empirical coefficients using their LES data base, and evaluate three new coefficients that MY2.5 does not have since their terms were neglected;
- Invent prescription of the length scale dependence on distance from the ground, PBL turbulence, buoyancy;
- Check the performance of the scheme in well-known experimental data (Businger et al., 1971; Wangara Day 33);

From Nakanishi and Niino (2009):

The good performance of the MYNN model relies partly on the improvement of the stability functions S_M and S_H for momentum and heat, respectively, through the parameterization of the pressure covariances that includes buoyancy effects (Figs. 1 and 2), and partly on the expression for the stability functions S_q , $S_{\theta l}$, $S_{\theta q}$, and S_{qw} for the third-order turbulent fluxes through S_M (Figs. 5 and 6). The major improvement in its performance, however, is due to our formulation of the turbulent length scale L that realistically increases with decreasing stability (Fig. 7).

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